MATHEMATICS

IMPORTANT FORMULAE / POINTS

1. RELATIONS AND FUNCTIONS

1. The Cartesian product of A and B is denoted by $A\times B$, i.e., $A\times B=\{(a,b)\,/\,a\in A,\ b\in B\}$
2. $A\times B\neq B\times A$
3. $n(A\times B)=n(B\times A)$
4. $A\times B=\phi$, if and only if $A=\phi$ or $B=\phi$
5. If $n(A)=p$ and $n(B)=q$, then $n(A\times B)=pq$
6. If $n(A)=p$ and $n(B)=q$, then the total number of relations that exist between A and B is 2^{pq}
7. Distributive property of Cartesian product
   (i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$
   (ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$
8. A Relation R from A to B is always a subset of $A\times B$. ie., $R\subseteq A\times B$.
9. If $f:x\rightarrow y$ is a **function** only if every element in the domain of f has an image.
10. The set of all first elements in the ordered pair is called **Domain.**
11. The set of all second elements in the ordered pair is called **Co-domain.**
12. The **range** of a function is a subset of co-domain.
13. If $n(A)=p$ and $n(B)=q$, then the total number of functions that exist between A and B is q^{p}.

2. NUMBERS AND SEQUENCES

1. Euclid's division lemma: If $'a'$ and $'b'$ are two positive integers $(a>b)$, then there exist unique integers $'q'$ and $'r'$ such that $a=bq+r,\ 0\leq r<b$ (Where $'q'$ is the quotient and $'r'$ is the remainder.
   (Note: The remainder $'r'$ is always less than the divisor $'b'$)
2. Every composite number can be expressed uniquely as the product of powers of prime.
   $(e.g.)\quad (32760=2^{3}\times3^{2}\times5^{1}\times7^{1}\times13^{1})$
3. Two positive integers are said to be relatively prime or co-prime if their HCF is 1.
4. Two integers $'a'$ and $'b'$ are said to be congruent modulo m, written as $a\equiv b(\text{mod } m)$, if they leave same remainder when divided by m. $a\equiv b(\text{mod } m)\Rightarrow a-b=mk$, for some integer k.

Arithmetic Progression. (A.P)

5. The n^{th} term of an A.P is $t_n = a + (n-1)d$
6. No. of terms $n = \frac{l-a}{d} + 1$
7. If the three numbers a, b, c are in A.P., then $2b = a + c$.
8. Sum to 'n' terms of an A.P

   (i) $S_n = \frac{n}{2}[2a + (n-1)d]$

   (ii) $S_n = \frac{n}{2}[a + l]$

9. $t_n = S_n - S_{n-1}$

Geometric Progression (G.P.)

10. The n^{th} term of a G.P. is $t_n = ar^{n-1}$
11. If three numbers a, b, c are in G.P. then $b^2 = ac$.
12. Sum to 'n' terms of a G.P.

    (i) $S_n = \frac{a(r^n - 1)}{r-1}$, if $r \neq 1$.

    (ii) $S_n = na$, if $r = 1$

13. Sum to infinite terms of a G.P. is $S_\infty = \frac{a}{1-r}$, if $-1 < r < 1$

Special Series

14. Sum of first 'n' natural numbers $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$.
15. Sum of first 'n' odd natural numbers $= n^2$
16. Sum of first 'n' odd natural numbers, if last term 'l' is given is $\left(\frac{l+1}{2}\right)^2$.
17. Sum of squares of first 'n' natural numbers $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
18. Sum of cubes of first 'n' natural numbers $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

3. ALGEBRA

1. A linear equation in three variables of the form $ax+by+cz+d=0$ represents a plane.
2. The product of two polynomials is the product of their LCM and GCD i.e. $f(x)\times g(x)=LCM\times GCD$.
3. $LCM=\dfrac{f(x)\times g(x)}{GCD}$.
4. The general form of Quadratic Equation is $ax^2+bx+c=0$
5. The roots of Quadratic Equation are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
6. (i) Sum of the roots $\alpha+\beta=-\dfrac{b}{a}$

   (ii) Product of roots $\alpha\beta=\dfrac{c}{a}$
7. If the roots are given, then the required quadratic equation is of the form $x^2-(SOR)x+POR=0$
8. Nature of roots of a quadratic equation.

Sl.No.	Discriminant $\Delta=b^2-4ac$	Nature of roots
(i)	$\Delta>0$	Real and unequal
(ii)	$\Delta=0$	Real and equal
(iii)	$\Delta<0$	not real

9. (i) $a^2-b^2=(a+b)(a-b)$

   (ii) $a^3+b^3=(a+b)(a^2-ab+b^2)$

   (ii) $a^3-b^3=(a-b)(a^2+ab+b^2)$
10. $x^4+x^2y^2+y^4=(x^2+xy+y^2)(x^2-xy+y^2)$
11. $x^4+x^2+1=(x^2+x+1)(x^2-x+1)$
12. $(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$
13. (i) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

    (ii) $\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

    (iii) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

    (iv) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$

    (v) $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

    (vi) $\alpha^4-\beta^4=(\alpha+\beta)(\alpha-\beta)(\alpha^2+\beta^2)$

14. $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

15. A matrix is a rectangular array of elements arranged in rows and columns.

16. A matrix having m rows and n columns is of the order $m \times n$

17. If the order of matrix A is $m \times n$ and B is $n \times p$, then the order of product matrix AB is $m \times p$

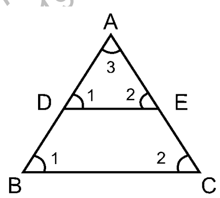
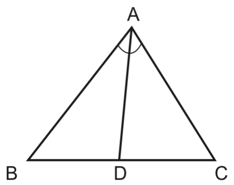
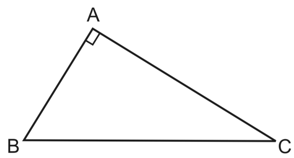
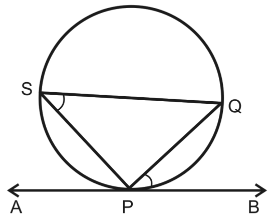
A	B	AB
$m \times n$	$n \times p$	$m \times p$

(Note: The column of first matrix and rows of the second matrix are equal)

18. Reversal Law of Transpose of matrix is $(AB)^T = B^T A^T$

4. GEOMETRY

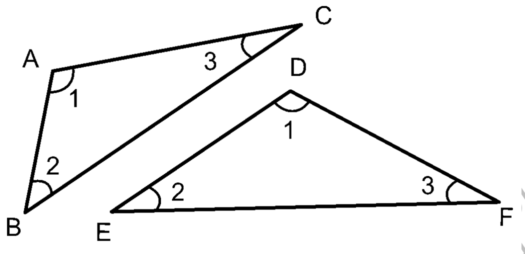
THEOREMS

Sl.No.	Statement	Diagram	
1.	**Basic Proportionality Theorem (Thales Theorem)** If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.		$\dfrac{AD}{DB} = \dfrac{AE}{EC}$
2.	**Angle Bisector Theorem** The internal bisector of an angle of triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.		$\dfrac{AB}{AC} = \dfrac{BD}{DC}$
3.	**Pythagoras Theorem (Baudhyana Theorem)** In a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.		$BC^2 = AB^2 + AC^2$
4.	**Alternate Segment Theorem** If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.		$\angle QPB = \angle PSQ$

CONCURRENCY THEOREMS

Theorems	Statement
1. Ceva's Theorem	Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC}\times\frac{CE}{EA}\times\frac{AF}{FB}=1$
2. Menelaus Theorem	A necessary and sufficient conditions for the points P,Q,R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC}\times\frac{CQ}{QA}\times\frac{AR}{RB}=-1$

SIMILAR TRIANGLES



If two triangles are similar

(i) The ratio of corresponding sides are equal to the ratio of corresponding perimeters.

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{FD}=\frac{AB+BC+CA}{DE+EF+FD}$$

(ii) The ratio of areas of two similar triangles are equal to the ratio of squares of the corresponding sides.

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{FD^2}$$

* The length of two tangents drawn from an exterior point to a circle are equal.
* If two circles touch externally, the distance between their centres is equal to the sum of their radii i.e., $OP = r_1 + r_2$

5. COORDINATE GEOMETRY

1. The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2. Mid point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
3. (i) The point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$

   internally in the ratio $m:n$ is $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

   (ii) The point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$

   externally in the ratio $m:n$ is $\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$

4. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$= \frac{1}{2}\begin{pmatrix} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{pmatrix} \text{ sq. units.}$$

(OR)

$$= \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\} \text{ sq.units}$$

5. Condition for three points are collinear is

(i) Area of $\Delta ABC = 0$ (or) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

(ii) Slope of AB = Slope of BC.

6. The area of the quadrilateral formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ is

$$= \frac{1}{2}\begin{bmatrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{bmatrix} \text{ sq. units}$$

(OR)

$$= \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\right\} \text{ sq. units}$$

7. Slope $m = \tan\theta$ (when inclination θ is given)

8. Slope of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

9. Slope of line $ax + by + c = 0$ is $m = -\dfrac{\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-a}{b}$

10. Equation of straight lines

Sl.No.	FORM	EQUATION
(i)	General equation of straight line	$ax + by + c = 0$
(ii)	Slope m, $y-$ intercept c	$y = mx + c$
(iii)	Slope m, point (x_1, y_1)	$y - y_1 = m(x - x_1)$
(iv)	Two points (x_1, y_1), (x_2, y_2)	$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$
(v)	x - intercept a, $y-$ intercept b	$\dfrac{x}{a} + \dfrac{y}{b} = 1$

11. Two lines are parallel, if and only if their slopes are equal. i.e., $m_1 = m_2$

12. Two lines are perpendicular then product of their slopes is $= -1$ i.e. $m_1 \times m_2 = -1$

13. If the slopes of both the pairs of opposite sides are equal, then the quadrilateral is a parallelogram.

14. If slope of $AB \times \text{Slope of } AC = -1,$ then AB is perpendicular to AC, $\angle A = 90^\circ$ and ΔABC is a right angled triangle.

15. Formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32.$

16. Equation of straight line parallel to the line $ax + by + c = 0$ is of the form $ax + by + k = 0$.

17. Equation of straight line perpendicular to the line $ax + by + c = 0$ is of the form $bx - ay + k = 0$

6. TRIGONOMETRY

1. Trigonometric Ratios

(i)	$sin\theta = \frac{opposite\ side}{hypotenuse}$	(viii)	$\cos ec\theta = \frac{1}{\sin\theta}$
(ii)	$cos\theta = \frac{adj.side}{hypotenuse}$	(ix)	$\cos\theta = \frac{1}{sec\theta}$
(iii)	$\tan\theta = \frac{opp.side}{adj.side}$	(x)	$\sec\theta = \frac{1}{\cos\theta}$
(iv)	$\cos ec\theta = \frac{\text{hypotenuse}}{\text{opposite}}$	(xi)	$\tan\theta = \frac{1}{\cot\theta}$
(v)	$\sec\theta = \frac{\text{hypotenuse}}{\text{adj.side}}$	(xii)	$\cot\theta = \frac{1}{\tan\theta}$
(vi)	$\cot\theta = \frac{\text{adj.side}}{\text{opp.side}}$	(xiii)	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
(vii)	$\sin\theta = \frac{1}{\cos ec\theta}$	(xiv)	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

2. Trigonometric Ratios for Complementary Angles

(i)	$sin(90^\circ - \theta) = \cos\theta$	(iv)	$\cos ec(90^\circ - \theta) = \sec\theta$
(ii)	$\cos(90^\circ - \theta) = \sin\theta$	(v)	$sec(90^\circ - \theta) = \cos ec\theta$
(iii)	$\tan(90^\circ - \theta) = \cot\theta$	(vi)	$\cot(90^\circ - \theta) = \tan\theta$

3. Trigonometric Ratios for $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90°

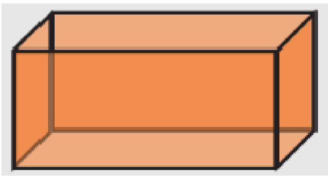
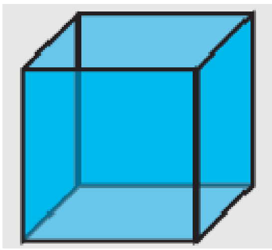
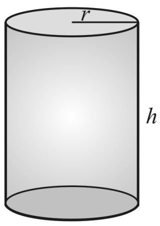
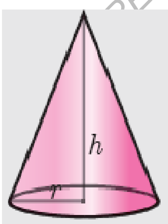
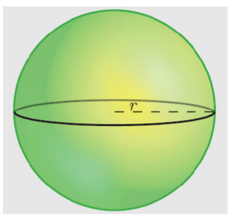
Trig. Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

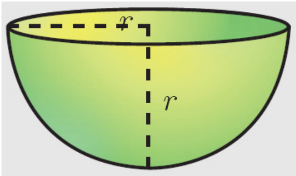
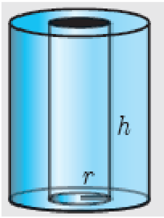
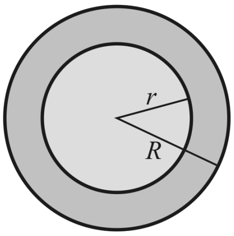
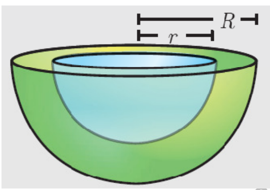
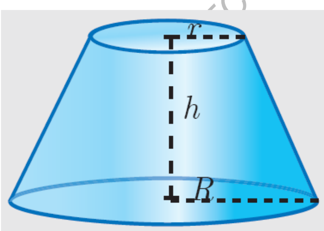
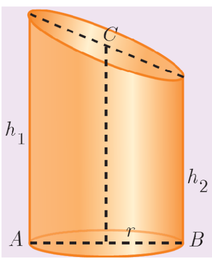
4. Trigonometric Identities

(i)	$\sin^2\theta + \cos^2\theta = 1$	(vi)	$\sec^2\theta - \tan^2\theta = 1$
(ii)	$\sin^2\theta = 1 - \cos^2\theta$	(vii)	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$
(iii)	$\cos^2\theta = 1 - \sin^2\theta$	(viii)	$\cot^2\theta = \operatorname{cosec}^2\theta - 1$
(iv)	$1 + \tan^2\theta = \sec^2\theta$	(ix)	$\operatorname{cosec}^2\theta - \cot^2\theta = 1$
(v)	$\tan^2\theta = \sec^2\theta - 1$		

5. The angle of elevation and angle of depression are equal.
6. The angle of elevation and depression are usually measured by device called <u>Clinometers.</u>

7. MENSURATION

Solid / Figure	Curved surface Area / Lateral surface Area (in sq. units)	Total surface Area (in sq. units)	Volume (in cubic units)
Cuboid 	$2h(l+b)$	$2(lb+bh+lh)$	$l \times b \times h$
Cube 	$4a^2$	$6a^2$	a^3
Right Circular Cylinder 	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
Right Circular Cone 	πrl $l=\sqrt{r^2+h^2}$ $l=$ slant height	$\pi rl+\pi r^2$ $=\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Hemisphere 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Hollow cylinder 	$2\pi(R+r)h$	$2\pi(R+r)$ $(R-r+h)$	$\pi(R^2-r^2)h$
Hollow sphere 	$4\pi R^2$ = outer surface area	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
Hollow hemisphere 	$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
Frustum of right circular cone 	$\pi(R+r)l$ where $l=\sqrt{h^2+(R-r)^2}$	$\pi(R+r)l+\pi R^2 + \pi r^2$	$\frac{1}{3}\pi h[R^2+r^2+Rr]$
Oblique frustum 	$2\pi r\times\frac{h_1+h_2}{2}$	--	$\pi r^2\times\frac{h_1+h_2}{2}$

8. STATISTICS AND PROBABILITY

1. Arithmetic mean, $\overline{x} = \dfrac{\sum x}{n}$

2. Range is the difference between the largest value and the smallest value i.e. $R = L - S$

3. Coefficient of Range $= \dfrac{L - S}{L + S}$

4. Variance $\sigma^2 = \dfrac{\Sigma(x - \overline{x})^2}{n}$

5. (i) $\sum x = n\overline{x}$

   (ii) $\sum (x - \overline{x}) = 0$

6. <u>Standard deviation for an Ungrouped data</u>

   (i) $\sigma = \sqrt{\dfrac{\sum d^2}{n}}$ where $d = x - \overline{x}$ and $\overline{x}$ is the mean.

   (ii) $\sigma = \sqrt{\dfrac{\sum d^2}{n} - \left(\dfrac{\sum d}{n}\right)^2}$ where $d = x - A$ and A is the assumed mean.

7. <u>Standard deviation for a Grouped data</u>

   (i) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f}}$ where $d = x - \overline{x}$ and $\overline{x}$ is the mean.

   (ii) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f} - \left(\dfrac{\sum fd}{\sum f}\right)^2}$ where $d = x - A$ and A is the assumed mean.

8. The standard deviation remains unchanged, if each value of the data is increased or decreased.

9. Standard deviation gets multiplied / divided, if each value is multiplied / divided by a fixed constant.

10. Standard deviation $\sigma = \sqrt{\text{Variance}}$

11. Standard deviation of first 'n' natural numbers $\sigma = \sqrt{\dfrac{n^2 - 1}{12}}$

12. Variance of first 'n' natural numbers $\sigma^2 = \dfrac{n^2 - 1}{12}$

13. Coefficient of variation, $\text{C.V.} = \frac{\sigma}{\overline{x}} \times 100\%$

14. The set of all possible outcomes is called sample space.

15. Each element of a sample space is called sample point.

16. The probability of an event E is $P(E) = \frac{n(E)}{n(S)}$

17. (i) Probability of a sure event is 1.

(ii) Probability of an impossible event is 0.

18. Probability value always lies from 0 to 1. i.e., $0 \le P(E) \le 1$

19. The probability of complement of an event E is $P(\overline{E}) = 1 - P(E)$

20. Addition Theorem of probability.

(i) $P(\text{A} \cup \text{B}) = P(A) + P(B) - P(A \cap B)$

(ii) $P(\text{A} \cup \text{B} \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

21. If A and B are mutually exclusive events then $P(\text{A} \cup \text{B}) = P(A) + P(B)$

22. (i) $P(\text{only } A) = P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(ii) $P(\text{only } B) = P(\overline{A} \cap B) = P(B) - P(A \cap B)$

23. (i) No. of days in a leap year = 366 days (52 weeks + 2 days)

(ii) No. of days in a non-leap year = 365 days (52 weeks + 1 day)

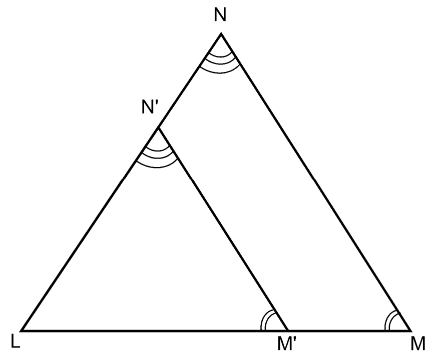
8. MARKS QUESTION

GEOMETRY

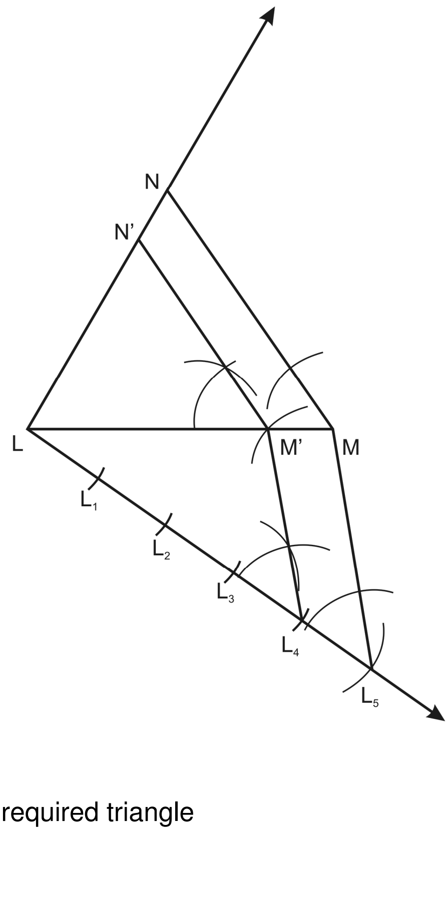
1. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle ($Scale\ factor \frac{4}{5} < 1$)

<u>Given:</u> Δ LMN

Rough Diagram



Fair diagram

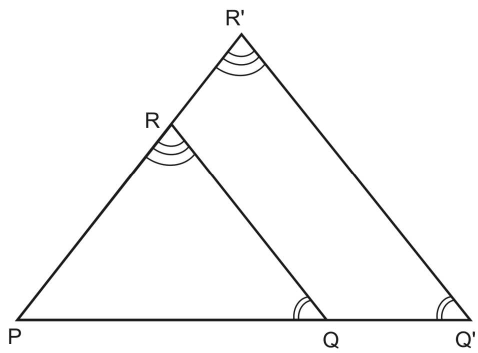


$LM'N'$ is the required triangle

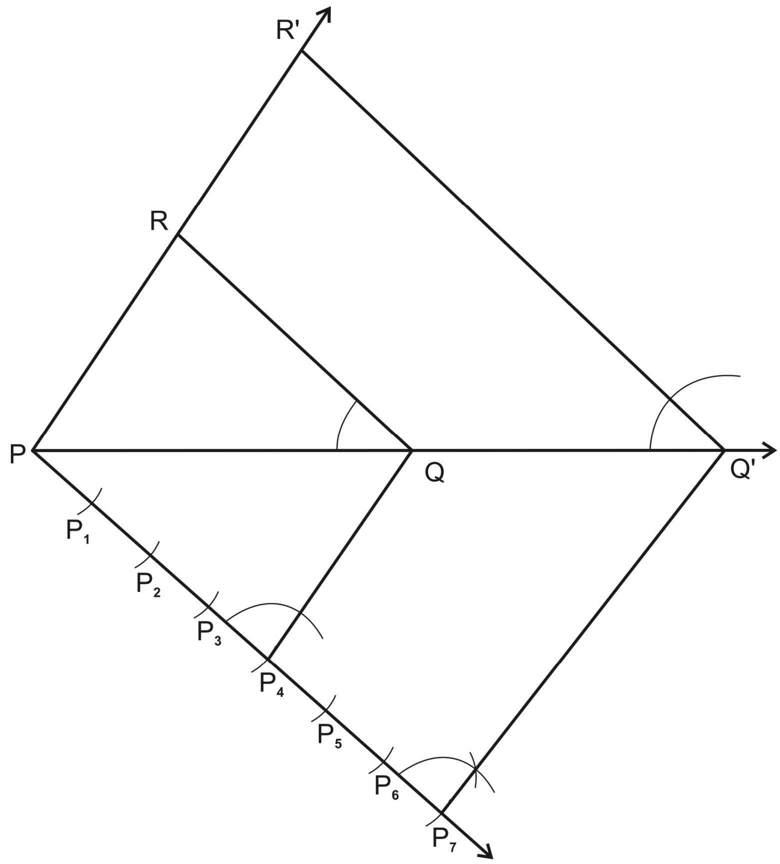
2. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR ($Scale\ factor\ \frac{7}{4} > 1$)

<u>Given:</u> Δ PQR

Rough Diagram



Fair diagram



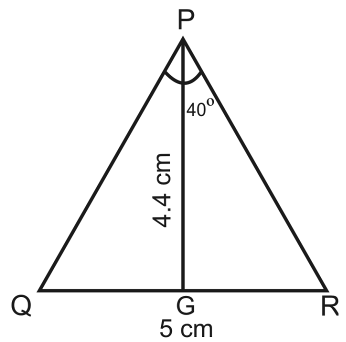
3. Construct a ΔPQR in which $QR = 5\,cm$, $\angle P = 40^\circ$ and the median PG from P to QR is $4.4\ cm$. Find the length of the altitude from P to QR.

Given: $QR = 5$ cm

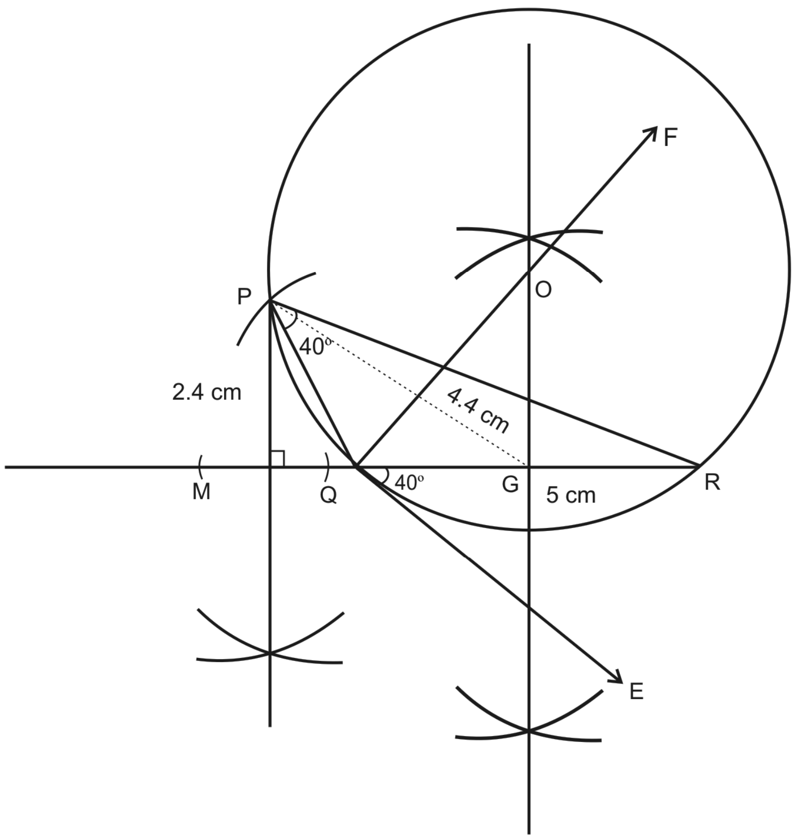
$\angle P = 40^\circ$

Median $PG = 4.4\ cm.$

Rough Diagram



Fair diagram



Length of the altitude from P to QR is $2.4\ cm.$

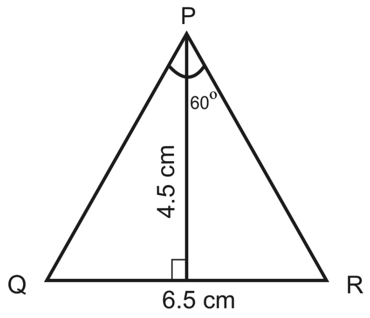
4. Construct a ΔPQR such that $QR = 6.5 cm, \angle P = 60^\circ$ and the altitude from P to QR is of length $4.5\ cm$.

Given: $QR = 6.5$ cm

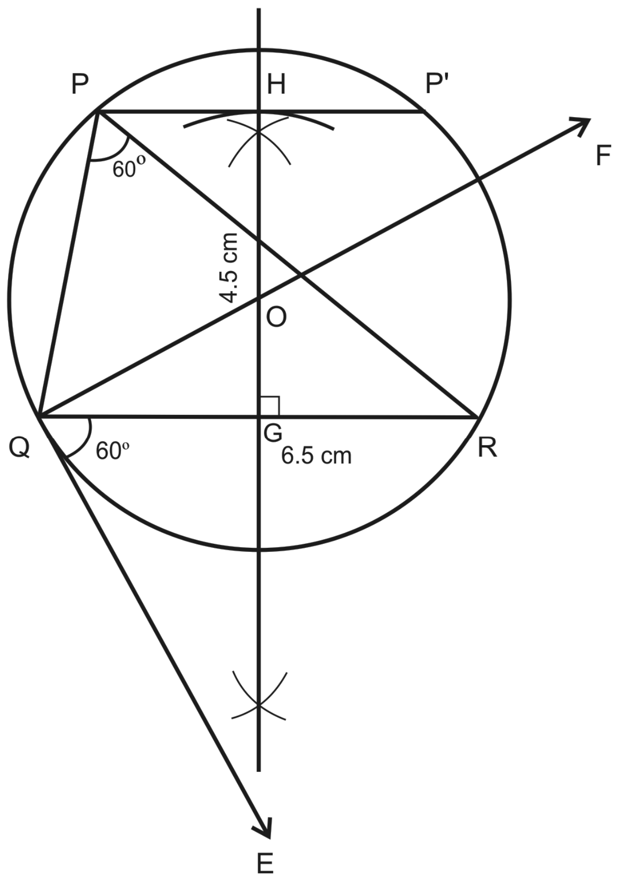
$\angle P = 60^\circ$

altitude from $P = 4.5\ cm$

Rough Diagram



Fair diagram



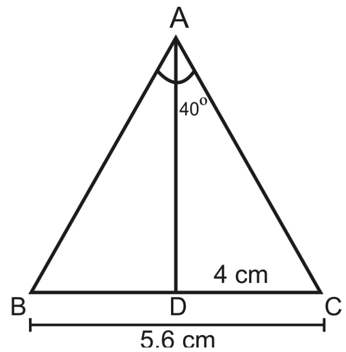
5. Draw a triangle ABC of base $BC = 5.6cm$, $\angle A = 40^\circ$ and the bisector of $\angle A$ meets BC at D such that $CD = 4cm.$

Given: $BC = 5.6cm$

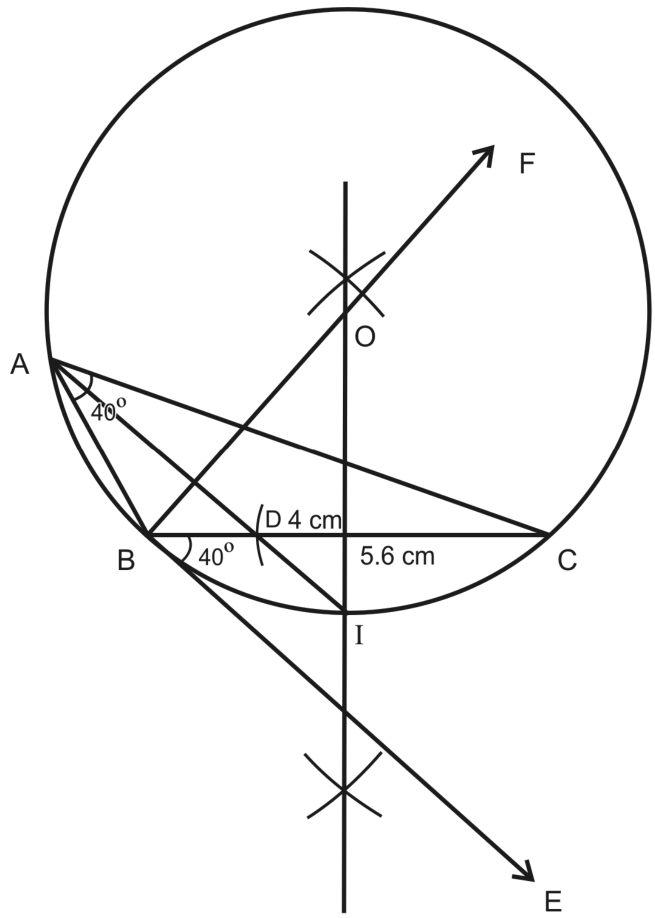
$\angle A = 40^\circ$

bisector of $\angle A$ meets $CD = 4cm$

Rough Diagram



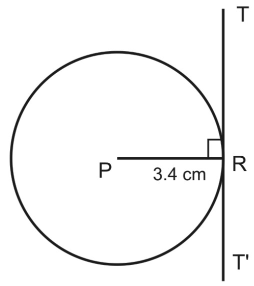
Fair diagram



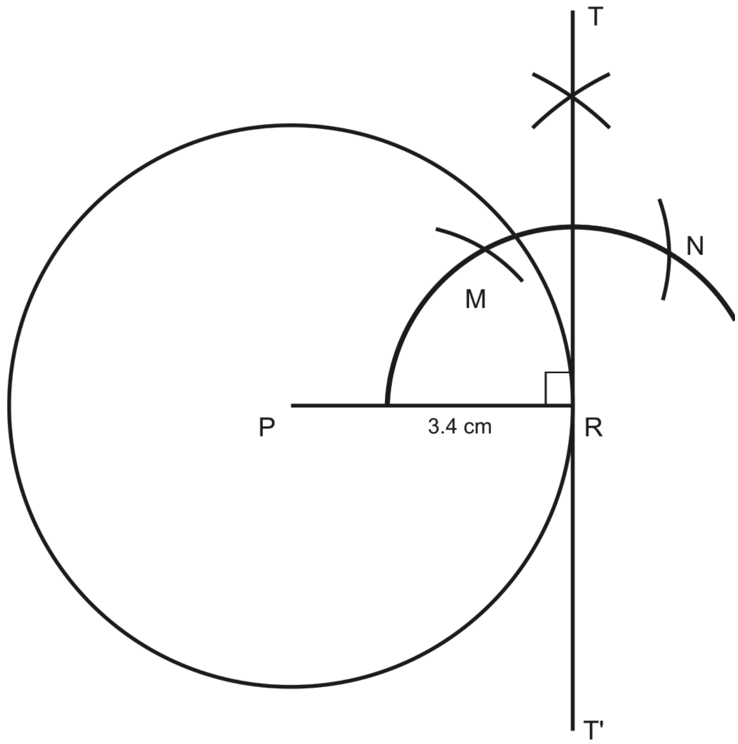
6. Draw a tangent at any point R on the circle of radius $3.4\,cm$ and centre at P.

Given: $radius = 3.4\,cm$

Rough Diagram



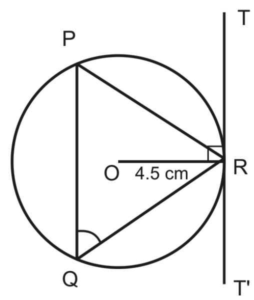
Fair diagram



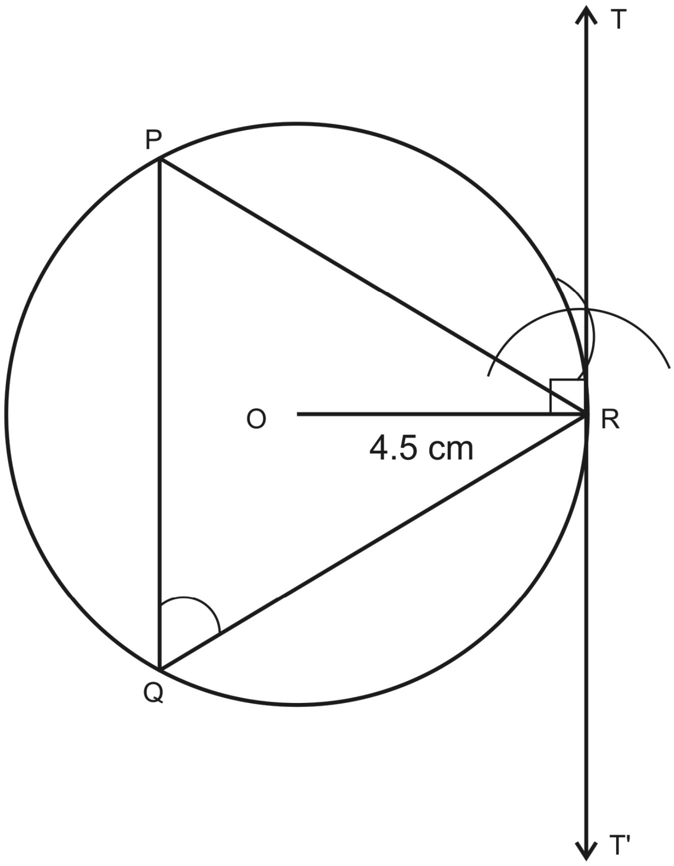
7. Draw a circle of radius $4.5\,cm.$ Take a point on the circle. Draw tangent at that point using the alternate segment theorem.

Given: $radius = 4.5\,cm$

Rough Diagram



Fair diagram



8. Draw a circle of diameter $6\,cm$ from a point P, which is $8\,cm$ away from its centre. Draw the two tangents PA and PB to the circle and measure its length.

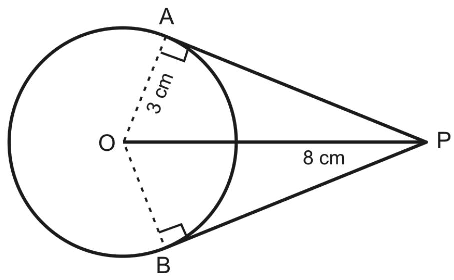
Given:

Diameter $= 6\,cm$

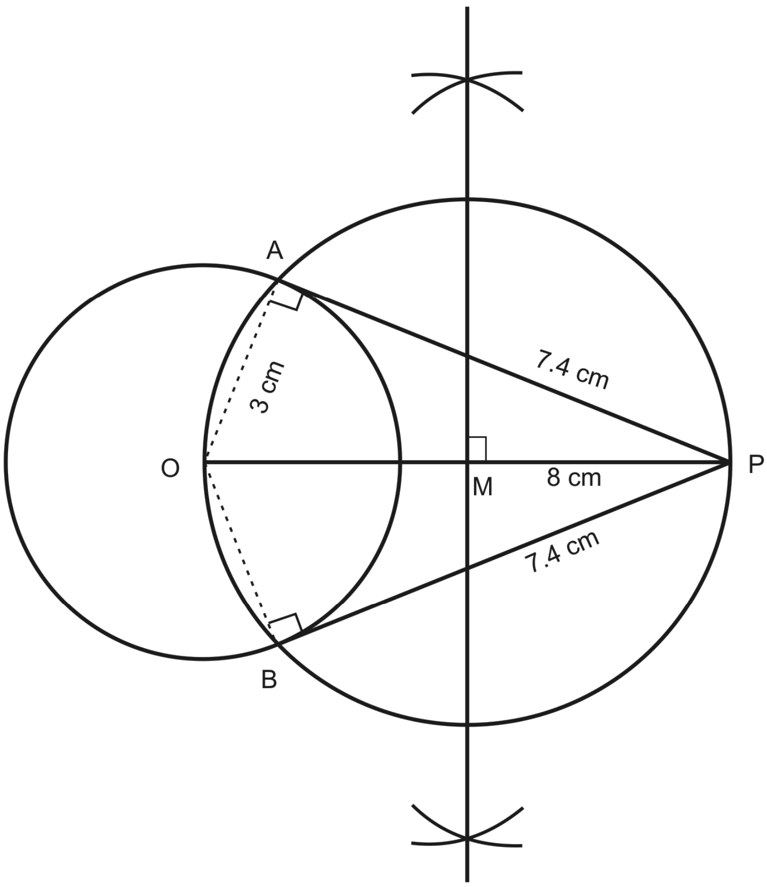
radius $= 3\,cm$

Distance $= 8\,cm$

Rough Diagram



Fair diagram



Length of tangent $PA = 7.4\,cm$

$PB = 7.4\,cm$

9. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR $\left(Scale\ factor\ \frac{3}{5}<1\right)$

10. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR $\left(Scale\ factor\ \frac{2}{3}<1\right)$

11. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR $\left(Scale\ factor\ \frac{7}{3}>1\right)$

12. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC $\left(Scale\ factor\ \frac{6}{5}>1\right)$

13. Construct a ΔPQR in which $PQ = 8\text{cm}$, $\angle R = 60^\circ$ and the median RG from R to PQ is 5.8cm. Find the length of the altitude from R to PQ.

14. Draw a triangle PQR in which the base $PQ = 4.5\text{cm}$, $\angle R = 35^\circ$ and the median from R to PQ is $6cm$.

15. Construct a ΔPQR such that $QR = 5\text{cm}$, $\angle P = 30^\circ$ and the altitude from P to QR is the length 4.2cm.

16. Construct a ΔABC such that $AB = 5.5cm$, $\angle C = 25^\circ$ and the altitude from C to AB is 4cm.

17. Draw a triangle ABC in which the base $BC = 8cm$, $\angle A = 60^\circ$ and the bisector of $\angle A$ meets BC at D such that $BD = 6\text{cm}$.

18. Draw ΔPQR such that $PQ = 6.8\text{cm}$, vertical angle is 50° and the bisector of vertical angle meets the base at D where $PD = 5.2\text{cm}$.

19. Draw a circle of radius 3cm. Take a point P on this circle and draw a tangent at P.

20. Draw a circle of radius 4cm. At a point L on it, draw a tangent to the circle using the alternate segment theorem.

21. Draw two tangents from a point which is 10cm away from the centre of a circle of radius 5cm. Also, measure the lengths of the tangents.

22. Draw two tangents from a point which is 5cm away from the centre of a circle of diameter 6cm. Also, measure the lengths of the tangents.

23. Take a point which is 11cm away from the centre of a circle of radius 4cm and draw the two tangents to the circle from that point.

24. Draw a tangent to the circle from the point P having radius 3.6 cm and centre at O. Point P is at a distance 7.2cm from the centre.

GRAPH

1. Draw the graph of the quadratic equation $x^2 - 4x + 4 = 0$ and state its nature of solution.

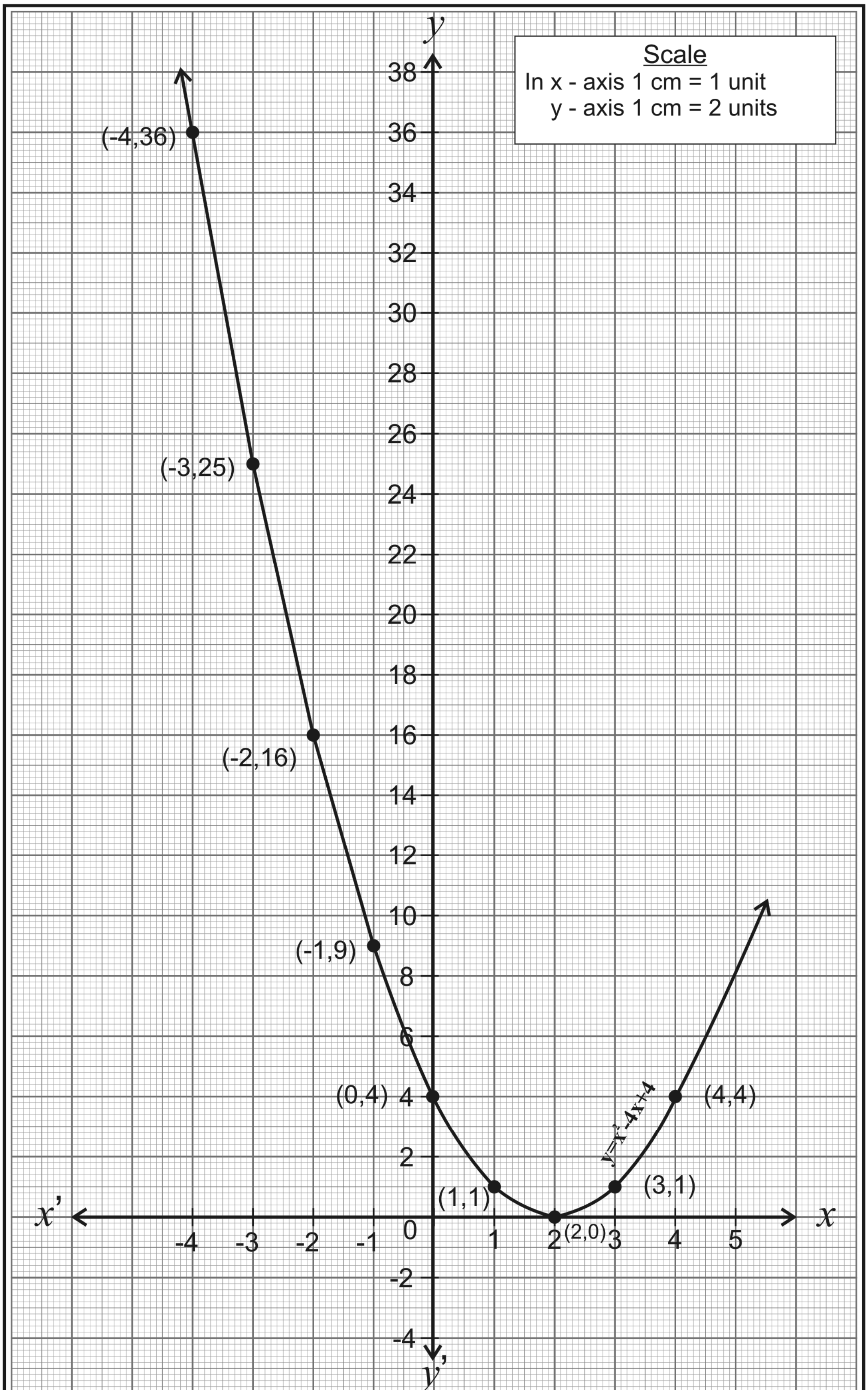
Step 1: Draw the graph of $y = x^2 - 4x + 4$

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	–4	–8	–12	–16
4	4	4	4	4	4	4	4	4	4
$y = x^2 - 4x + 4$	36	25	16	9	4	1	0	1	4

The points are $(-4,36)$, $(-3,25)$, $(-2,16)$, $(-1,9)$, $(0,4)$, $(1,1)$, $(2,0)$, $(3,1)$ and $(4,4)$

RESULT:

The roots are real and equal.


y
Scale
In x - axis 1 cm = 1 unit
y - axis 1 cm = 2 units
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
(-4,36)
(-3,25)
(-2,16)
(-1,9)
(0,4)
(1,1)
(2,0)
(3,1)
(4,4)
y=x²-4x+4
x'
x
y'
-4
-3
-2
-1
1
2
3
4
5

2. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$

Step 1: Draw the graph of $y = 2x^2$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y = 2x^2$	32	18	8	2	0	2	8	18	32

The points are $(-4,32)$, $(-3,18)$, $(-2,8)$, $(-1,2)$, $(0,0)$, $(1,2)$, $(2,8)$, $(3,18)$ and $(4,32)$

Step 2: By solving, we get

$$y = \cancel{2x^2} \qquad \rightarrow (1)$$

$$0 = \overset{(-)}{\cancel{2x^2}} \overset{(+)}{-} x \overset{(+)}{-} 6 \qquad \rightarrow (2)$$

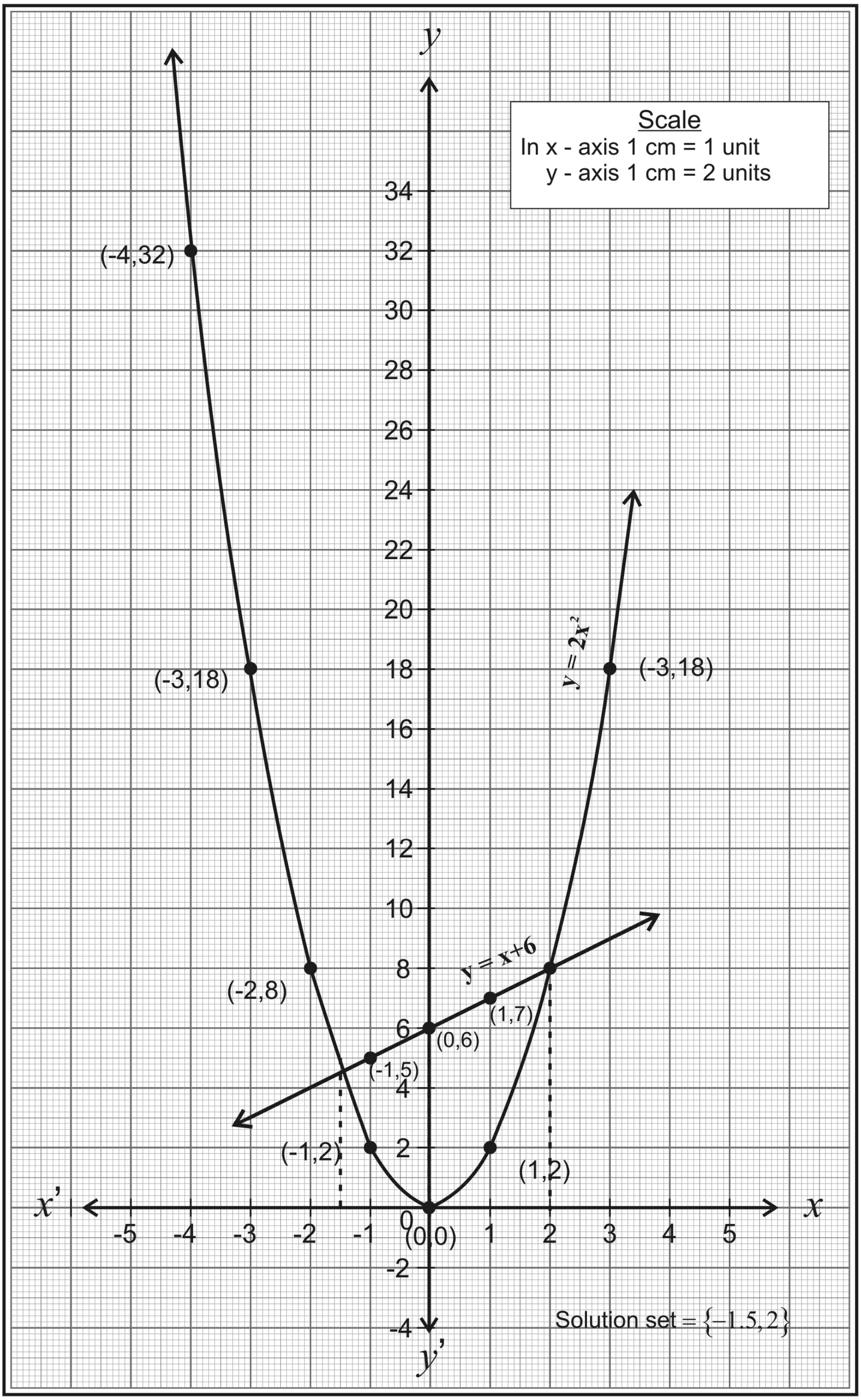
$$(1) - (2) \Rightarrow \quad y = x + 6$$

x	-1	0	1
6	6	6	6
$y = x + 6$	5	6	7

The points are $(-1,5)$, $(0,6)$ and $(1,7)$

RESULT:

Solution set $= \{-1.5, 2\}$


Scale
In x - axis 1 cm = 1 unit
y - axis 1 cm = 2 units
y
x
x'
y'
(-4,32)
(-3,18)
(-3,18)
(-2,8)
(-1,2)
(1,2)
(-1,5)
(0,6)
(1,7)
(0,0)
y = 2x²
y = x+6
Solution set = {−1.5, 2}

3. Draw the graph of the quadratic equation $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

Step 1: Draw the graph of $y = x^2 + x - 2$

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	–4	–3	–2	–1	0	1	2	3	4
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
$y = x^2 + x - 2$	10	4	0	–2	–2	0	4	10	18

The points are $(-4,10), (-3,4), (-2,0), (-1,-2), (0,-2), (1,0), (2,4), (3,10), (4,18)$

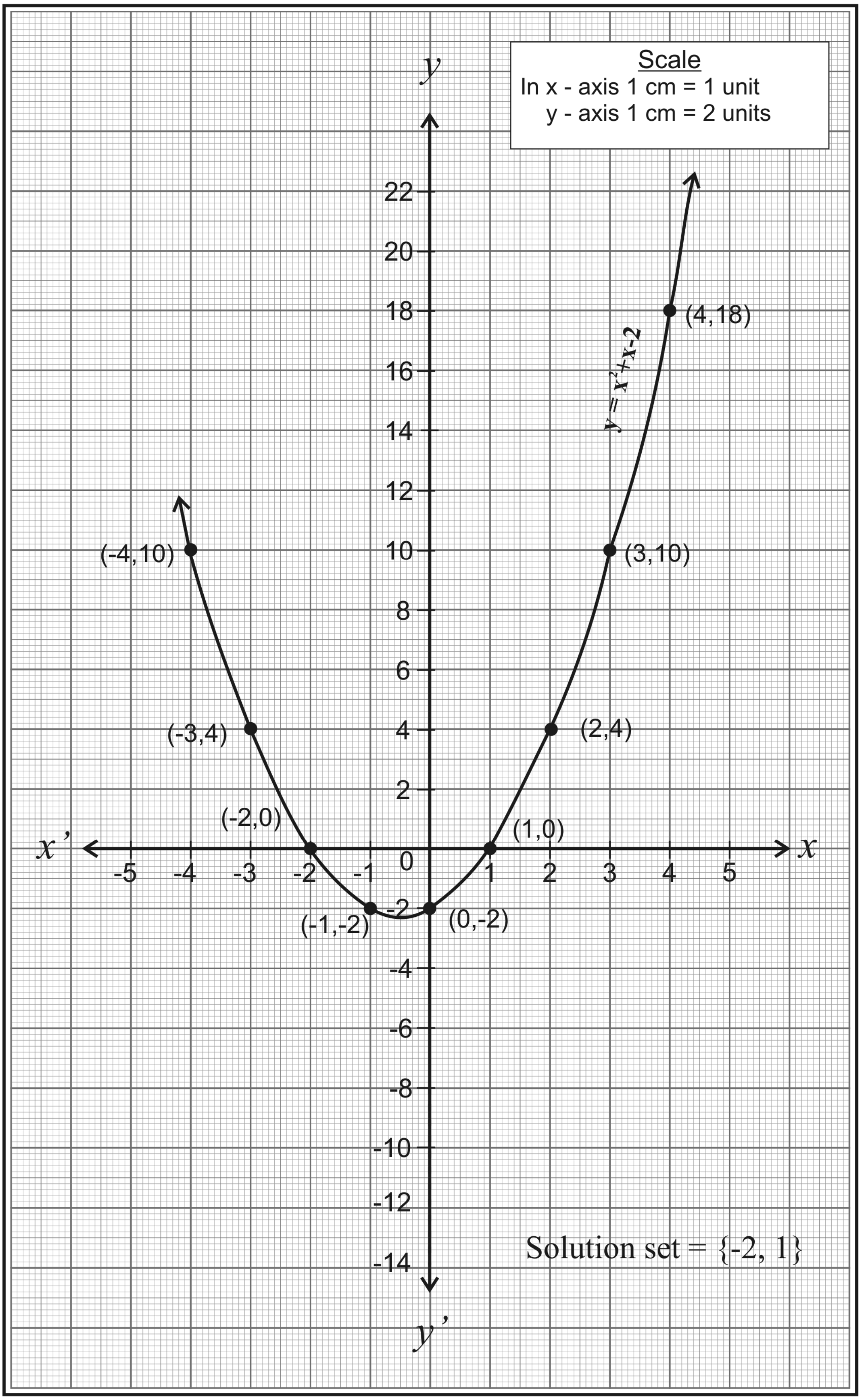
Step 2: By solving, we get

$$\begin{array}{rrcccl} & y = & \not{x^2} & +\not{x} & \not{-2} & \rightarrow (1) \\ & 0 = & \overset{(+)}{\not{x^2}} & \overset{(-)}{+\not{x}} & \overset{(+)}{-\not{2}} & \rightarrow (2) \\ \hline (1)-(2) \Rightarrow & y = & 0 & & & \\ \hline \end{array}$$

$y = 0$ represents $x - axis$

RESULT:

Solution set $= \{-2, 1\}$


Scale
In x - axis 1 cm = 1 unit
y - axis 1 cm = 2 units
y
x
x'
y'
$y = x^2+x-2$
(4,18)
(3,10)
(-4,10)
(2,4)
(-3,4)
(-2,0)
(1,0)
(-1,-2)
(0,-2)
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-5
-4
-3
-2
-1
1
2
3
4
5
Solution set = {-2, 1}

4. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.

Step 1: Draw the graph of $y = x^2 + x$

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	–4	–3	–2	–1	0	1	2	3	4
$y = x^2 + x$	12	6	2	0	0	2	6	12	20

The points are $(-4,12), (-3,6), (-2,2), (-1,0), (0,0), (1,2), (2,6), (3,12), (4,20)$

Step 2: By solving, we get

$$\begin{array}{rrcccl} & y = & x^2 & + \; x & & \rightarrow (1) \\ & 0 = & \overset{(-)}{x^2} & \overset{(-)}{+} \; 0x & \overset{(-)}{+} \; 1 & \rightarrow (2) \\ \hline (1)-(2) \Rightarrow & y = & & x & -\,1 & \end{array}$$

x	–1	0	1
–1	–1	–1	–1
$y = x - 1$	–2	–1	0

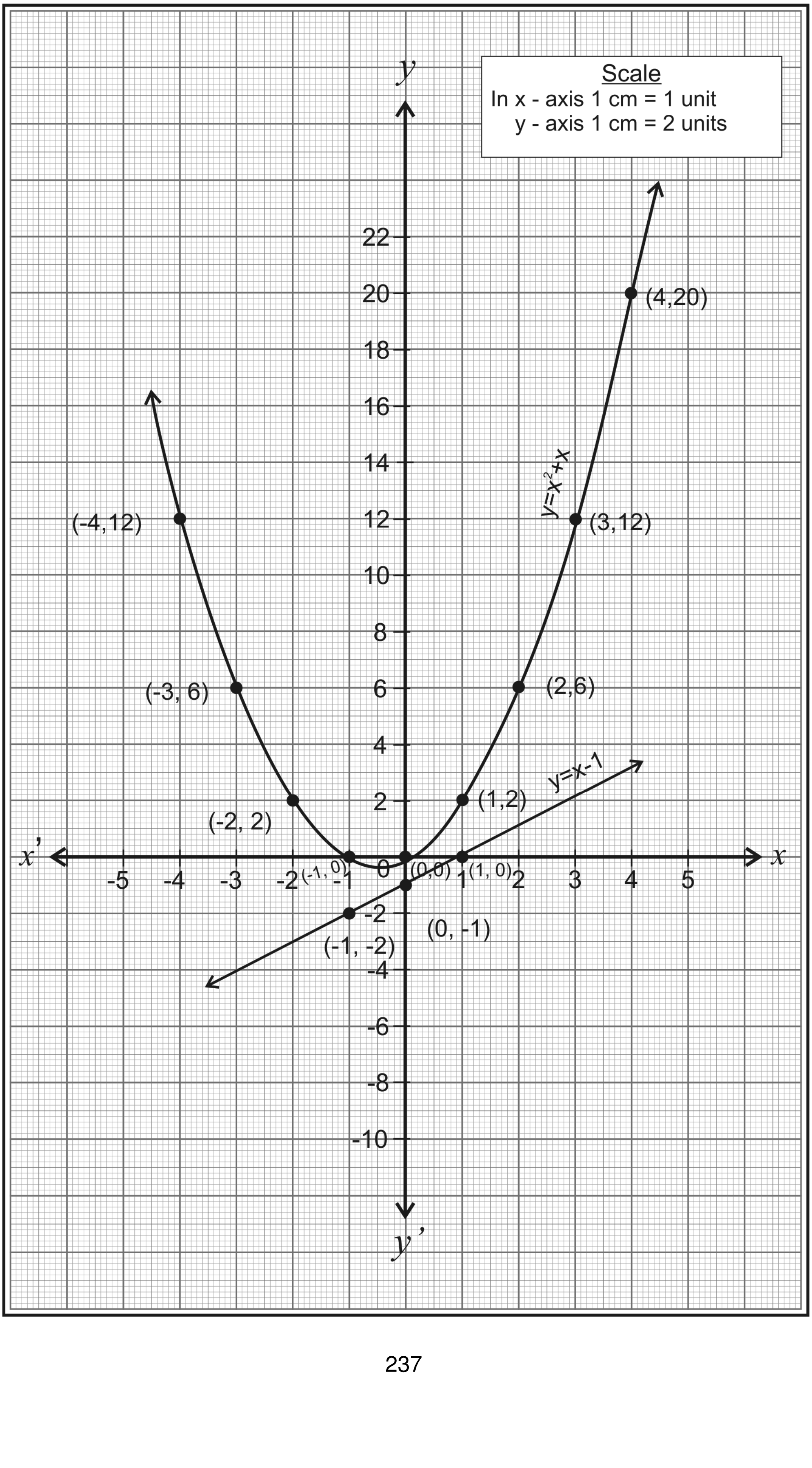
The points are $(-1,-2)$ $(0,-1)$ and $(1,0)$

Step 3:

$y = x^2 + x$ and $y = x - 1$ does not intersect.

RESULT:

$x^2 + 1 = 0$ has no real roots


Scale
In x - axis 1 cm = 1 unit
y - axis 1 cm = 2 units
y
x'
x
y'
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
1
2
3
4
5
(4,20)
(-4,12)
(3,12)
(-3, 6)
(2,6)
(-2, 2)
(1,2)
(-1, 0)
(0,0)
(1, 0)
(-1, -2)
(0, -1)
y=x²+x
y=x-1

5. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.

Step 1 Draw the graph of $y = x^2 - 5x - 6$

x	–4	–3	–2	–1	0	1	2	3	4	5	6	7
x^2	16	9	4	1	0	1	4	9	16	25	36	49
$-5x$	20	15	10	5	0	–5	–10	–15	–20	–25	–30	–35
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = x^2 - 5x - 6$	30	18	8	0	–6	–10	–12	–12	–10	–6	0	8

The points are

$(-4,30), (-3,18), (-2,8), (-1,0), (0,-6), (1,-10), (2,-12), (3,-12), (4,-10), (5,-6), (6,0), (7,8)$

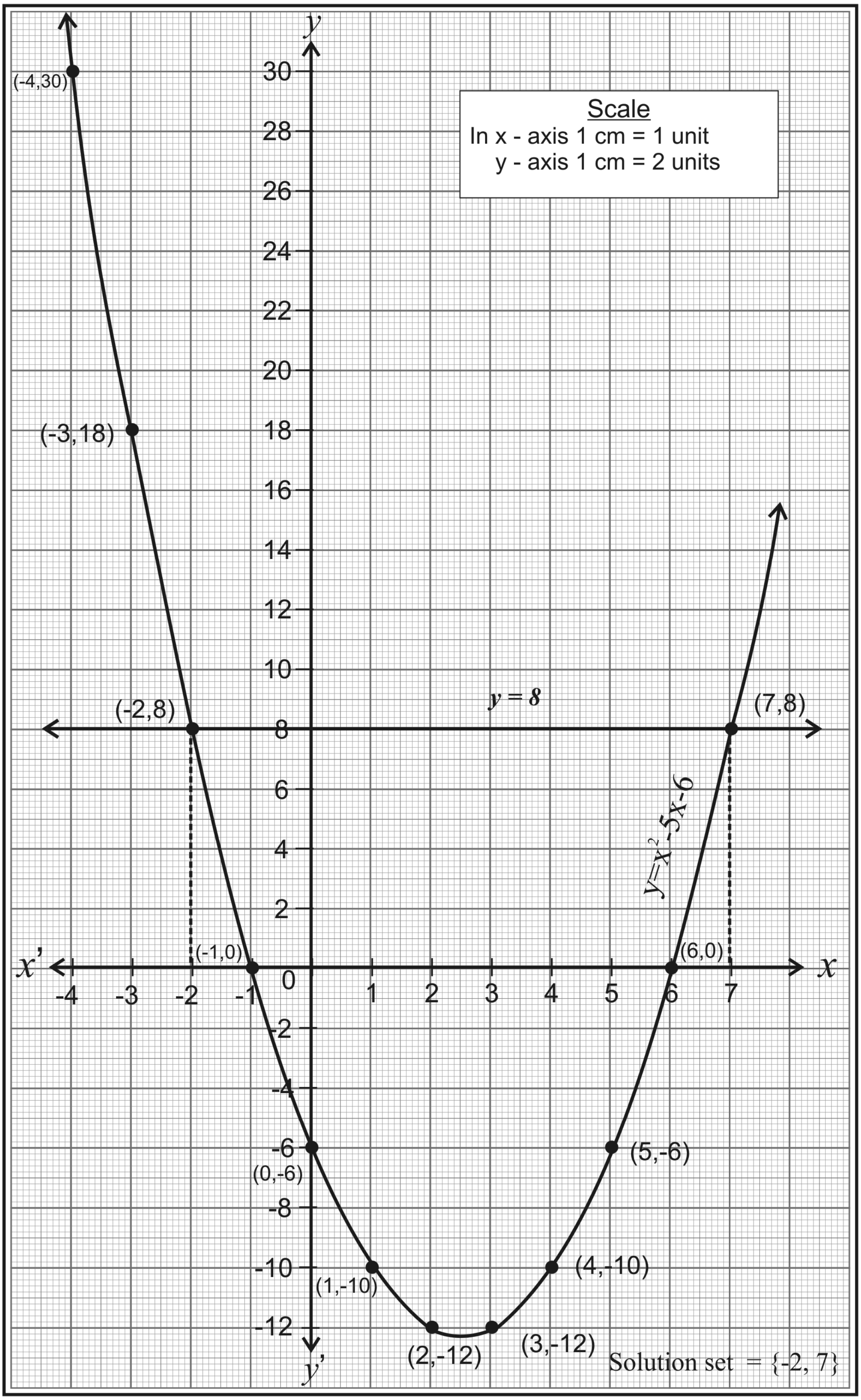
Step 2: By solving

$$\begin{array}{rrrrrl} & y = & x^2 & -5x & -6 & \rightarrow (1) \\ & 0 = & \overset{(-)}{x^2} & \overset{(+)}{-5x} & \overset{(+)}{-14} & \rightarrow (2) \\ \hline (1)-(2) \Rightarrow & y = & & & 8 & \\ \hline \end{array}$$

x	–2	–1	0	1	2
$y = 8$	8	8	8	8	8

RESULT:

Solution set $= \{-2, 7\}$


Scale
In x - axis 1 cm = 1 unit
y - axis 1 cm = 2 units
y
x
x'
y'
(-4,30)
(-3,18)
(-2,8)
(7,8)
y = 8
y=x²-5x-6
(-1,0)
(6,0)
(0,-6)
(5,-6)
(1,-10)
(4,-10)
(2,-12)
(3,-12)
Solution set = {-2, 7}

6. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$

(i) graph the equation
(ii) find the total MB of the song
(iii) after how many seconds will 75% of the song gets downloaded
(iv) after how many seconds the song will be downloads completely,

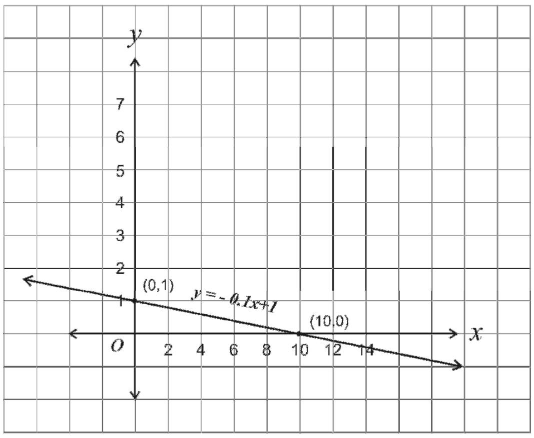
<u>(i)</u> <u>Given</u> $y = -0.1x + 1$ ....(1)

sub. $x = 0$, $y = -0.1(0) + 1$
$= 0 + 1$
$= 1$

sub $y = 0$ $0 = -0.1x + 1$
$0.1x = 1$
$\frac{1}{10}x = 1$
$\Rightarrow x = 10$

x	0	10
y	1	0

The points are $(0,1)$ and $(10,0)$



ii) The total MB is obtained by, sub $x = 0$ in the given equation (1), we get

i.e., $y = -0.1(0) + 1$
$= 0 + 1$
$\boxed{y = 1\text{MB}}$

Therefore, total MB of the song $= 1$ MB

iii) The time when 75% of song gets downloaded

remaining % $= 25$ i.e., $y = 25$

$$\therefore \text{sub } y = 25 \text{ in (1), we have}$$

$$\begin{aligned} y &= -0.1x+1 \\ 0.25 &= -0.1x+1 \\ 0.1x &= 1-0.25 \\ 0.1x &= 0.75 \\ \frac{1}{10}x &= \frac{75}{100} \\ x &= \frac{75}{100} \times 10 \\ &= 7.5 \end{aligned}$$

$\therefore 75\%$ of the song gets downloaded in 7.5 seconds.

iv) Song downloaded completely

remaining $\% = 0$ $(y = 0)$

$$\begin{aligned} \text{sub } y &= 0 \text{ in the equation} \\ y &= -0.1x+1 \\ 0 &= -0.1x+1 \\ 0.1\,x &= 1 \\ \frac{1}{10}\,x &= 1 \\ x &= 10 \end{aligned}$$

$\therefore$ No. of seconds required for downloading the song completely $= 10$ seconds.

7. Draw the graph of the quadratic equation $x^2 - 9 = 0$ and state its nature of solutions.
8. Draw the graph of the quadratic equation $(2x-3)(x+2) = 0$ and state its nature of solutions.
9. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$
10. Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$
11. Draw the graph of $y = x^2 - 4x + 3$ and hence use it to solve $x^2 - 6x + 9 = 0$
12. Draw the graph of $y = x^2 + x - 2$ and hence use it to solve $x^2 + 2x + 1 = 0$
13. Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$
14. Draw the graph of $y = 2x^2 - 3x - 5$ and hence use it to solve $2x^2 - 4x - 6 = 0$
15. Draw the graph of $y = (x-1)(x+3)$ and hence use it to solve $x^2 - x - 6 = 0$
16. A mobile phone is put to use when the battery power is 100%. The percent of battery power $'y'$ remaining after using the mobile phone for $'x'$ hours is assumed as $y = -0.25x + 1$.

    i) draw the graph of the equation.

    ii) Find the number of hours elapsed if the battery power is 40%

    iii) How much time does it takes so that the battery has no power?

5 MARK QUESTIONS

1. RELATIONS AND FUNCTIONS

1. Let $A = \{x \in W \mid x \le 2\}$, $B = \{x \in N \mid 1 < x \le 4\}$, and $C = \{3, 5\}$

verify $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Given

$A = \{x \in W \mid x \le 2\} \Rightarrow A = \{0,1,2\}$
$B = \{x \in N \mid 1 < x \le 4\} \Rightarrow B = \{2,3,4\}$
$C = \{3, 5\}$

LHS $A \times (B \cup C)$

$$B \cup C = \{2,3,4\} \cup \{3,5\} = \{2,3,4,5)$$

$$A \times (B \cup C) = \{0,1,2\} \times \{2,3,4,5\}$$
$$= \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4),(2,5)\} \quad ......(1)$$

RHS $(A \times B) \cup (A \times C)$

$$A \times B = \{0,1,2\} \times \{2,3,4\} = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4)\}$$

$$A \times C = \{0,1,2\} \times \{3,5\} = \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5)\}$$

$$(A \times B) \cup (A \times C) = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4),(2,5)\} \quad ....(2)$$

From (1) and (2), we see that, LHS = RHS

i.e. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence Verified.

2. Let $f : A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$ Represent f by

(i) an arrow diagram (ii) a set of ordered pairs
(iii) a table (iv) a graph

<u>Given</u> $A = \{2, 4, 6, 10, 12\}$

$B = \{0, 1, 2, 4, 5, 9\}$

$$f(x) = \frac{x}{2} - 1$$

$$f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$$

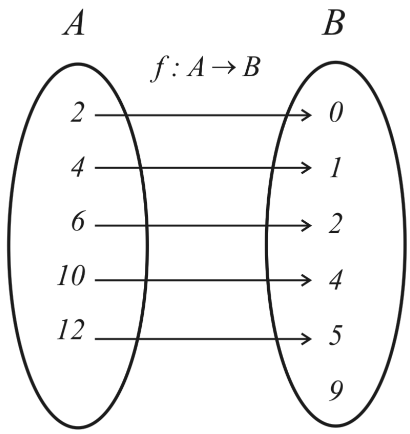
$$f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$$

$$f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$$

$$f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$$

$$f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$$

(i) <u>An arrow diagram</u>



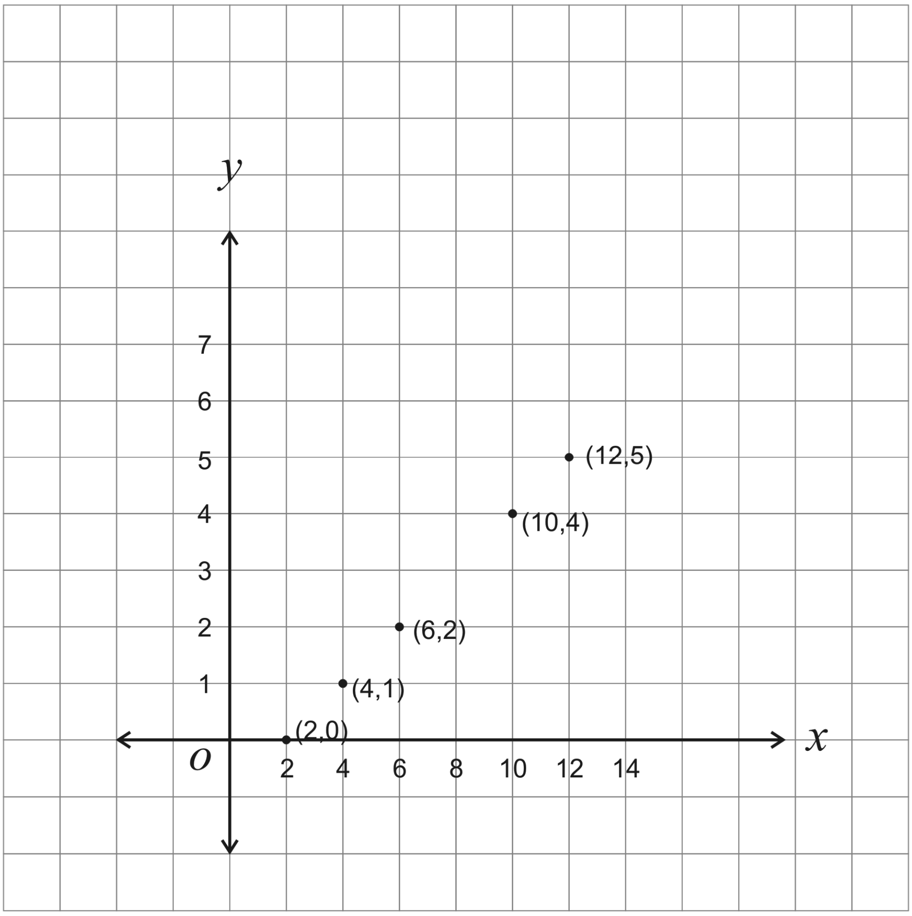
(ii) <u>Set of ordered pairs</u>

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

(iii) Table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iv) Graph



3. A function $f:[-5,9]\rightarrow R$ is defined as follows:-

$$f(x)=\begin{cases}6x+1; & -5\leq x<2\\ 5x^2-1; & 2\leq x<6\\ 3x-4; & 6\leq x\leq 9\end{cases}$$

Find (i) $f(-3)+f(2)$ (ii) $f(7)-f(1)$

(iii) $2f(4)+f(8)$ (iv) $\dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$

$f(-3)$	$f(-2)$	$f(1)$	$f(2)$
$f(x)=6x+1$ $f(-3)=6(-3)+1$ $=(-18)+1$ $=-17$	$f(x)=6x+1$ $f(-2)=6(-2)+1$ $=(-12)+1$ $=-11$	$f(x)=6x+1$ $f(1)=6(1)+1$ $=6+1$ $=7$	$f(x)=5x^2-1$ $f(2)=5(2)^2-1$ $=5(4)-1$ $=20-1$ $=19$
$f(4)$	$f(6)$	$f(7)$	$f(8)$
$f(x)=5x^2-1$ $f(4)=5(4)^2-1$ $=5(16)-1$ $=80-1$ $=79$	$f(x)=3x-4$ $f(6)=3(6)-4$ $=18-4$ $=14$	$f(x)=3x-4$ $f(7)=3(7)-4$ $=21-4$ $=17$	$f(x)=3x-4$ $f(8)=3(8)-4$ $=24-4$ $=20$

i) $f(-3)+f(2)=-17+19=2$

ii) $f(7)-f(1) = 17-7 = 10$

iii) $2f(4)+f(8) = 2(79)+20 = 158+20 = 178$

iv) $$\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{68} = \frac{-36}{68} = \frac{-9}{17}$$

4. If $f(x)=x-1, g(x)=3x+1$ and $h(x)=x^2$, then Show that $(fog)oh = fo(goh)$

Given

$f(x)=x-1$

$g(x)=3x+1$

$h(x)=x^2$

LHS $(fog)oh$

$$fog = f[g(x)]$$
$$= f(3x+1)$$
$$= 3x+1-1$$
$$= 3x$$
$$fog(oh) = fog[h(x)]$$
$$= fog(x^2)$$
$$= 3x^2 \qquad -(1)$$

RHS $fo(goh)$

$$
\begin{aligned}
goh &= g[h(x)] \\
&= g(x^2) \\
&= 3x^2+1 \\
fo(goh) &= f(goh) \\
&= f(3x^2+1) \\
&= 3x^2 \cancel{+1} \cancel{-1} \\
&= 3x^2 \qquad -(2)
\end{aligned}
$$

From (1) and (2), we see that, $\text{LHS} = \text{RHS}$

i.e. $(fog)oh = fo(goh)$

Hence Verified.

Another Method

LHS $(fog)oh$

$$
\begin{aligned}
&= [(x-1)o(3x+1)]\,o\,x^2 \\
&= (3x \cancel{+1} \cancel{-1})\,o\,x^2 \\
&= (3x)\,o\,x^2 \\
&= 3x^2 \qquad -(1)
\end{aligned}
$$

RHS $fo(goh)$

$$
\begin{aligned}
&= (x-1)o[(3x+1)\,o\,x^2] \\
&= (x-1)o(3x^2+1) \\
&= 3x^2 \cancel{+1} \cancel{-1} \\
&= 3x^2 \qquad -(2)
\end{aligned}
$$

From (1) and (2), we see that, $\text{LHS} = \text{RHS}$

i.e. $(fog)oh = fo(goh)$

Hence Verified.

5. Let $A = \{\text{The set of all natural numbers less than } 8\}$,
$B = \{\text{The set of all prime numbers less than } 8\}$
$C = \{\text{The set of even prime numbers}\}$

Verify $A \times (B - C) = (A \times B) - (A \times C)$

6. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f : A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function f as

(i) an arrow diagram (ii) a set of ordered pairs
(iii) a table form and (iv) a graphical form

7. If $f : R \to R$ is defined as follows

$$f(x) = \begin{cases} 2x + 7; & x < -2 \\ x^2 - 2; & -2 \le x < 3 \\ 3x - 1; & x \ge 3 \end{cases}$$

Find (i) $f(4)$ (ii) $f(-2)$ (iii) $f(4) + 2f(1)$

(iv) $\dfrac{f(1) - 3f(4)}{f(-3)}$

8. A function $f : R \to R$ is defined as follows

$$f(x) = \begin{cases} x + 2, & x > 1 \\ 2, & -1 \le x \le 1 \\ x - 1, & -3 < x < -1 \end{cases}$$

Find (i) $f(0)$ (ii) $f(-1.5)$ (iii) $f(2) + f(-2)$

9. Let f be a function defined by $f(x) = 3x + 2; x \in N$

i) Find the images of 1, 2, 3

ii) Find the pre-images of 29, 53

iii) Identify the type of function.

10. If $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$ Show that $(fog)oh = fo(goh)$

11. If $f(x) = x^2, g(x) = 3x$ and $h(x) = x - 2$ Show that $(fog)oh = fo(goh)$

2. NUMBERS AND SEQUENCES

1. Find the greatest number consisting of 6 digits which is exactly divisible by $24,\ 15,\ 36$?.

Solution: The greatest 6 digit number is 999999.

L.C.M of $24, 15, 36$ is 360.

2	24, 15, 36
2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

$$\begin{array}{r} 277 \\ 360\overline{)999999} \\ 720 \\ \hline 2799 \\ 2520 \\ \hline 2799 \\ 2520 \\ \hline 279 \end{array}$$

$$\begin{aligned} LCM &= 2\times2\times2\times3\times3\times5 \\ &= 360 \end{aligned}$$

Required greatest number consisting of 6 digits divisible by $24,\ 15,\ 36\ =\ 999999\ -\ 279$

$= 999720$

2. The sum of three consecutive terms that are in A.P is 27 and their product is 288. Find the three terms.

Solution:

Let the three terms of the A.P. be $a-d,\ a,\ a+d$

Given sum $=\ 27$

$$\Rightarrow a - \cancel{d} + a + a + \cancel{d} = 27$$

$$3a = 27$$

$$a = \frac{27}{3}$$

$$= 9$$

$$Product = 288$$

$$\Rightarrow (a-d)\ a\,(a+d) = 288$$

$$(9-d)\ 9\,(9+d) = 288$$

$$(9-d)\ (9+d) = \frac{\overset{32}{\cancel{288}}}{\cancel{9}}$$

$$9^2 - d^2 = 32$$
$$81 - d^2 = 32$$
$$-d^2 = 32 - 81$$
$$-d^2 = -49$$
$$d = \sqrt{49}$$
$$= \pm 7$$

When $a = 9, \quad d = 7$	$a = 9, \quad d = -7$
$a - d = 9 - 7 = 2$	$a - d = 9 - (-7)$
$a = 9$	$= 9 + 7 = 16$
$a + d = 9 + 7 = 16$	$a = 9$
	$a + d = 9 - 7 = 2$

$\therefore$The Three terms are $2,\ 9,\ 16\ (or)\ 16,\ 9,\ 2$

3. Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

$$300 \quad \frac{sum}{divisible\ by\ 7} \quad 600$$

$$7\overline{)300+1}\ (42 \qquad 28,\ 20,\ 14,\ 6+1$$

$$7\overline{)600-5}\ (85 \qquad 56,\ 40,\ 35,\ 5$$

$a = 300 + 1 = 301$, $\quad d = 7$, $\quad l = 600 - 5 = 595$

$$n = \frac{l - a}{d} + 1$$
$$= \frac{595 - 301}{7} + 1$$
$$= \frac{294}{7} + 1 \quad (42)$$
$$= 42 + 1$$
$$n = 43$$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{43}{2}(301+595)$$

$$= \frac{43}{\cancel{2}}(\cancel{896}^{448})$$

$$= 19264$$

$\therefore$ Sum of all the natural numbers between 300 and 600 which are divisible by 7 is 19264.

4. In a G.P. the 9th term is 32805 and 6th term is 1215. Find the 12th term.

Given:

$$t_9 = 32805$$

$$t_6 = 1215$$

$$t_{12} = ?$$

$$t_9 = 32805 \Rightarrow ar^8 = 32805 \quad \text{....(1)}$$

$$t_6 = 1215 \Rightarrow ar^5 = 1215 \quad \text{....(2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\cancel{a}r^8}{\cancel{a}r^5} = \frac{\cancel{32805}^{27}}{\cancel{1215}_{1}}$$

$$r^3 = 27$$

$$r^3 = 3^3$$

$$\boxed{r = 3}$$

Sub $r = 3$ in (2), we get

$$ar^5 = 1215$$

$$a(3)^5 = 1215$$

$$a(243) = 1215$$

$$a = \frac{\cancel{1215}^{5}}{\cancel{243}_{1}}$$

$$\boxed{a = 5}$$

$$t_{12} = ar^{11}$$

$$= 5(3)^{11}$$

$$= 5 \times 177147$$

$$= 885735$$

$\therefore 12^{th}$ term is 885735.

5. Find the sum of the series $10^3+11^3+12^3+....+20^3$

$$\begin{aligned}10^3+11^3+12^3+........+20^3 &= (1^3+2^3+3^3+.......+20^3)-(1^3+2^3+3^3+.......+9^3)\\ &=\sum 20^3-\sum 9^3\\ &=\left[\frac{n(n+1)}{2}\right]^2_{n=20} - \left[\frac{n(n+1)}{2}\right]^2_{n=9}\\ &=\left(\frac{\overset{10}{\cancel{20}}\times 21}{\cancel{2}}\right)^2 - \left(\frac{9\times \overset{5}{\cancel{10}}}{\cancel{2}}\right)^2\\ &=(210)^2 - (45)^2\\ &=(210+45) - (210-45)\\ &=255\times 165\\ &=42075\end{aligned}$$

6. Find the sum to n terms of the series $3+33+333+..........\text{to } n \text{ terms}$

Solution:

$$\begin{aligned}S_n &= 3+33+333+..........\text{to } n\ terms\\ &=3\,(1+11+111+..........\text{to } n \text{ terms})\\ &=\frac{3}{9}[\,9+99+999+...........\text{to } n \text{ terms}\,] \qquad \text{(Multiply Nr./Dr. by 9)}\\ &=\frac{\cancel{3}}{\underset{3}{\cancel{9}}}[\,(10-1)+(100-1)+(1000-1)+...........\text{to } n \text{ terms}\,]\\ &=\frac{1}{3}[\,\underbrace{(10+100+1000+.......\text{to } n \text{ terms})}_{\text{G.P}}-(1+1+1+....\text{to } n \text{ terms})\,]\end{aligned}$$

$$a=10, \quad r=\frac{100}{10}=10>1 \qquad S_n=\frac{a(r^n-1)}{r-1}$$

$$\text{Required sum } S_n=\frac{1}{3}\left(\frac{10(10^n-1)}{10-1}-n\right)$$

$$=\frac{1}{3}\left(\frac{10(10^n-1)}{9}-n\right)$$

$$=\frac{10(10^n-1)}{27}-\frac{n}{3}$$

7. If a, b, c are three consecutive terms of a A.P and x, y, z are the three terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

Given:

a, b, c are 3 consecutive terms of an A.P.	x, y, z are 3 consecutive terms of a G.P
$\left.\begin{aligned} a &= a \\ b &= a+d \\ c &= a+2d \end{aligned}\right\} \quad -(1)$	$\left.\begin{aligned} x &= a \\ y &= ar \\ z &= ar^2 \end{aligned}\right\} \quad -(2)$

LHS: $x^{b-c} \times y^{c-a} \times z^{a-b}$

$$= x^{b-c} \times y^{c-a} \times z^{a-b}$$

$$= a^{a+d-(a+2d)} \times (ar)^{a+2d-a} \times (ar^2)^{a-(a+d)}$$

$$= a^{a+d-a-2d} \times (ar)^{a+2d-a} \times (ar^2)^{a-a-d}$$

$$= a^{-d} \times (ar)^{2d} \times (ar^2)^{-d}$$

$$= \frac{1}{a^d} \times a^{2d} \times r^{2d} \times a^{-d} \times r^{-2d}$$

$$= \frac{1}{a^d} \times a^{2d} \times r^{2d} \times \frac{1}{a^d} \times \frac{1}{r^{2d}}$$

$$= 1$$

=R.H.S.

Hence proved

8. Rekha has 15 square colour papers of sizes $10cm, 11cm, 12cm, ... 24cm.$ How much area can be decorated with these colour papers?

Area decorated with colour papers

$$= 10^2 + 11^2 + 12^2 + ... + 24^2$$

$$= (1^2 + 2^2 + 3^2 + ... + 24^2) - (1^2 + 2^2 + 3^2 + ... + 9^2)$$

$$= \sum 24^2 - \sum 9^2$$

$$= \left[\frac{n(n+1)(2n+1)}{6}\right]_{n=24} - \left[\frac{n(n+1)(2n+1)}{6}\right]_{n=9}$$

$$= \frac{\overset{4}{\cancel{24}} \times 25 \times 49}{\cancel{6}} - \frac{\overset{3}{\cancel{9}} \times \overset{5}{\cancel{10}} \times 19}{\underset{\cancel{2}}{\cancel{6}}}$$

$$= 4 \times 25 \times 49 - 3 \times 5 \times 19$$

$$= 10 \times 49 - 15 \times 19$$

$$= 4900 - 285$$

$$= 4615$$ ($\therefore$ Rekha can decorate 4615 cm^2 area with these colour papers.)

9. Find the HCF of $396,\ 504,\ 636$ using Euclid's division algorithm.

10. The l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that $(x-y)n+(y-z)l+(z-x)m=0$

11. In an A.P., sum of four consecutive term is 28 and sum of their squares is 276. Find the four numbers.

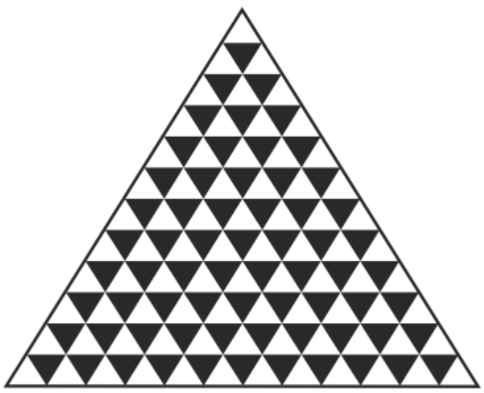
12. The sum of first $n, 2n$ and $3n$ terms of an A.P. are $S_1,\ S_2$ and S_3 respectively. Prove that $S_3=3(S_2-S_1)$

13. Find the sum $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+................to\ 12\ terms$

14. In an G.P., the product of three consecutive term is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms.

15. If $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....................\ n\ terms,$ then prove that $(x-y)S_n=\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}$

16. A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



3. ALGEBRA

1. Solve : $3x - 2y + z = 2$

$2x + 3y - z = 5$

$x + y + z = 6$

Given: The equations are

$$3x - 2y + z = 2 \qquad \text{---------(1)}$$
$$2x + 3y - z = 5 \qquad \text{---------(2)}$$
$$x + y + z = 6 \qquad \text{---------(3)}$$

Solve (1) and (2)

$$3x - 2y + \cancel{z} = 2 \qquad \text{---------(1)}$$
$$2x + 3y - \cancel{z} = 5 \qquad \text{---------(2)}$$
$$(1) + (2) \Rightarrow \quad 5x + y = 7 \qquad \text{---------(4)}$$

Solve (2) and (3)

$$2x + 3y - \cancel{z} = 5 \qquad \text{---------(2)}$$
$$x + y + \cancel{z} = 6 \qquad \text{---------(3)}$$
$$(2) + (3) \Rightarrow \quad 3x + 4y = 11 \qquad \text{---------(5)}$$

Solving (4) and (5), we get

$$(4) \times 4 \Rightarrow \quad 20x + 4y = 28 \qquad \text{---------(6)}$$
$$(5) \times 1 \Rightarrow \quad \overset{(-)}{3x} + \overset{(-)}{4y} = \overset{(-)}{11} \qquad \text{---------(5)}$$
$$(6) - (5) \Rightarrow \quad 17x = 17$$
$$x = \frac{17}{17}$$
$$\boxed{x = 1}$$

$Sub\ x=1\ \ in\ (4),\ \ we\ get$

$$5x+y=7$$
$$5(1)+y=7$$
$$5+y=7$$
$$y=7-5$$
$$\boxed{y=2}$$

$Sub\ \ x=1\ and\ \ y=2\ in\ (3),\ we\ get$

$$x+y+z=6$$
$$1+2+z=6$$
$$3+z=6$$
$$z=6-3$$
$$\boxed{z=3}$$

$\therefore x=1$

$y=2$

$z=3$

2. Find the GCD of $3x^4+6x^3-12x^2-24x$ and $4x^4+14x^3+8x^2-8x$

Let $f(x)=3x^4+6x^3-12x^2-24x$

$=3x(x^3+2x^2-4x-8)$

$g(x)=4x^4+14x^3+8x^2-8x$

$=2x(2x^3+7x^2+4x-4)$

GCD of $3x$ and $2x$ is x

Divide $g(x)$ by $f(x)$

$$\begin{array}{r|l} & 2 \\ \hline x^3+2x^2-4x-8 & 2x^3+7x^2+4x-4 \\ & \underset{(-)}{2x^3}\underset{(-)}{+4x^2}\underset{(+)}{-8x}\underset{(+)}{-16} \\ \hline & 3x^2+12x+12 \end{array}$$

$3(x^2+4x+4)\neq 0$

$$\begin{array}{r|l} & x-2 \\ \hline x^2+4x+4 & x^3+2x^2-4x-8 \\ & \underset{(-)}{x^3}\underset{(-)}{+4x^2}\underset{(-)}{+4x} \\ \hline & -2x^2-8x-8 \\ & \underset{(+)}{-2x^2}\underset{(+)}{-8x}\underset{(+)}{-8} \\ \hline & 0 \end{array}$$

$\therefore GCD\ =x(x^2+4x+4)$

3. If $9x^4+12x^3+28x^2+ax+b$ is a perfect square, find the values of a and b.

	$3x^2+2x+4$
$3x^2$	$9x^4+12x^3+28x^2+ax+b$
	$(-)\ 9x^4$
$6x^2+2x$	$12x^3+28x^2$
	$(-)\ 12x^3\ (-)\ +4x^2$
$6x^2+4x+4$	$24x^2+ax+b$
	$(-)\ 24x^2\ (-)\ +16x\ (-)\ +16$
	0

Since the given polynomial is a perfect square, remainder $=0$

$\Rightarrow a=16$ and $b=16$

4. Find the values of a and b, if the polynomial $4x^4-12x^3+37x^2+bx+a$ is a perfect square.

	$2x^2-3x+7$
$2x^2$	$4x^4-12x^3+37x^2+bx+a$
	$(-)\ 4x^4$
$4x^2-3x$	$-12x^3+37x^2$
	$(+)\ -12x^3\ (-)\ +9x^2$
$4x^2-6x+7$	$28x^2+bx+a$
	$(-)\ 28x^2\ (+)\ -42x\ (-)\ +49$
	0

Since the given polynomial is a perfect square, remainder $=0$

i.e $b+42=0$	$a-49=0$
$\Rightarrow b=-42$	$\Rightarrow a=49$

$\boxed{a=49}$ and $\boxed{b=-42}$

5. Find the square root of the expression $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}$

$$\frac{x}{y}-5+\frac{y}{x}$$

$$\begin{array}{r|l}
\frac{x}{y} & \frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2} \\
 & \overset{(-)}{}\ \frac{x^2}{y^2} \\ \hline
\frac{2x}{y}-5 & -\frac{10x}{y}+27 \\
 & \overset{(+)}{-}\frac{10x}{y}\ \overset{(-)}{+}25 \\ \hline
\frac{2x}{y}-10+\frac{y}{x} & 2-\frac{10y}{x}+\frac{y^2}{x^2} \\
 & \overset{(-)}{2}\ \overset{(+)}{-}\frac{10y}{x}\ \overset{(-)}{+}\frac{y^2}{x^2} \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}} = \left|\frac{x}{y}-5+\frac{y}{x}\right|$$

6. If $A=\begin{bmatrix}5 & 2 & 9\\ 1 & 2 & 8\end{bmatrix}$ and $B=\begin{bmatrix}1 & 7\\ 1 & 2\\ 5 & -1\end{bmatrix}$, Verify that $(AB)^T=B^TA^T$

Given: $A=\begin{bmatrix}5 & 2 & 9\\ 1 & 2 & 8\end{bmatrix}$

$$B=\begin{bmatrix}1 & 7\\ 1 & 2\\ 5 & -1\end{bmatrix}$$

$L.H.S \quad (AB)^T$

$$AB = \begin{bmatrix} 5 & \overrightarrow{2} & 9 \\ 1 & 2 & 8 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{pmatrix} (5 \quad 2 \quad 9)\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} & (5 \quad 2 \quad 9)\begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} \\ (1 \quad 2 \quad 8)\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} & (1 \quad 2 \quad 8)\begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \qquad \text{....(1)}$$

$R.H.S \quad B^T A^T$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}$$

$$A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$$

$$= \begin{pmatrix} (1 \quad 1 \quad 5)\begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & (1 \quad 1 \quad 5)\begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} \\ (7 \quad 2 \quad -1)\begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & (7 \quad 2 \quad -1)\begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \qquad \text{....(2)}$$

From (1) and (2), we see that

$(AB)^T = B^T A^T$

Hence verified

7. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I_2 = O$.

<u>Given</u> $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

<u>LHS</u> $A^2 - 5A + 7I_2$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{pmatrix} (3 \quad 1)\begin{pmatrix} 3 \\ -1 \end{pmatrix} & (3 \quad 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ (-1 \quad 2)\begin{pmatrix} 3 \\ -1 \end{pmatrix} & (-1 \quad 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$=\begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$A^2=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A=5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$=\begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix}$$

$$7I_2=7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$=\begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$A^2-5A+7I_2=\begin{bmatrix} 8 & 5 \\ -1 & 3 \end{bmatrix}-\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & 5 \\ -1 & 3 \end{bmatrix}+\begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$=O$$

= RHS

Hence showed

8. Find the values of a, b, c, d from the equation. $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$

Given $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$

By Equating, we get

$$a-b = 1 \quad -(1)$$
$$2a-b = 0 \quad -(2)$$
$$2a+c = 5 \quad -(3)$$
$$3c+d = 2 \quad -(4)$$

From (1), $a-b = 1 \Rightarrow a = 1+b$ ....(5)

From (2), $2a-b = 0 \Rightarrow 2a = b$ ....(6)

Sub $b = 2a$ in (5), we get

$$a = 1+2a$$
$$a-2a = 1$$
$$-a = 1$$
$$a = -1$$

Sub $a = -1$ in (6), we get

$$2(-1) = b$$
$$-2 = b$$
$$b = -2$$

Sub $a = -1$ in (3), we get

$$2a+c = 5$$
$$2(-1)+c = 5$$
$$-2+c = 5$$
$$c = 5+2$$
$$c = 7$$

Sub $c = 7$ in (4), we get

$$3c+d = 2$$
$$3(7)+d = 2$$
$$21+d = 2$$
$$d = 2-21$$
$$d = -19$$

$$\boxed{\therefore a = -1, \quad b = -2, \quad c = 7 \text{ and } d = -19}$$

9. The sum of the digits of a three digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digits is equal to the units digits then find the original three digit number.

10. Find the G.C.D of $(x^3+y^3), (x^4+x^2y^2+y^4)$ whose L.C.M is $(x^3+y^3), (x^2+xy+y^2)$

11. If $A=\frac{2x+1}{2x-1}, B=\frac{2x-1}{2x+1} \text{find} \frac{1}{A-B}-\frac{2B}{A^2-B^2}$

12. Find the square root of $64x^4-16x^3+17x^2-2x+1$

13. Find the values of 'm' and 'n' if $\frac{1}{x^4}-\frac{6}{x^3}+\frac{13}{x^2}+\frac{m}{x}+n$ is a perfect square.

14. The hypotenuse of a right angled triangle is 25cm and its perimeter is 56cm. Find the length of the smallest side.

15. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

16. There is a square field whose side is 10m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying flower bed and graveling the path at ₹3 and ₹4 per sq.m respectively is ₹364. Find the width of the gravel path.

17. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

18. If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ are real and equal, Prove that either $a=0\,(or)\,a^3+b^3+c^3=3abc$.

19. If the roots of $(a-b)x^2+(b-c)x+(c-a)=0$ are real and equal, then prove that b, a, c are in A.P.

20. If α and β are the roots of $x^2+7x+10=0$, find the values of i) $\alpha-\beta$, ii) $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$.

21. If α and β are the roots of $7x^2+ax+2=0$ and if $\beta-\alpha=-\frac{13}{7}$ find the values of 'a'

22. Solve $\sqrt{y+1}+\sqrt{2y-5}=3$.

23. If one root of the equation $3x^2+kx+81=0$ is the square of the other, then find the values of 'k'.

24. Given $A=\begin{pmatrix}1 & 3\\ 5 & -1\end{pmatrix}$, $B=\begin{pmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{pmatrix}$ and $C=\begin{pmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{pmatrix}$. Verify $A(B+C)=AB+AC$.

25. If $A=\begin{bmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{bmatrix}$, Show that $(AB)^T = B^T A^T$

26. If $A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$. Show that $(AB)C = A(BC)$.

27. Solve $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}4\\ 5\end{pmatrix}$

28. If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. Show that $A^2-(a+d)A=(bc-ad)I_2$

29. Find x and y if $x\begin{bmatrix}4\\ -3\end{bmatrix}+y\begin{bmatrix}-2\\ 3\end{bmatrix}=\begin{bmatrix}4\\ 6\end{bmatrix}$

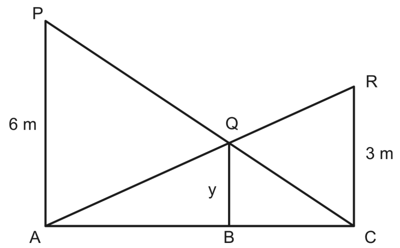
30. Let $A=\begin{pmatrix}2 & 0\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ and $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$. Show that $A(BC)=(AB)C$.

31. If α and β are the roots of x^2-2x+3. Find the polynomial whose roots are

i) $\alpha+2,\ \beta+2$ ii) $\dfrac{\alpha-1}{\alpha+1},\dfrac{\beta-1}{\beta+1}$.

4. GEOMETRY

1. Two vertical poles of heights $6\,\mathrm{m}$ and $3\ \mathrm{m}$ are erected above a horizontal ground *AC*. Find the value of y.



2. Two poles of height $'a'$ metres and $'b'$ metres are $'p'$ metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\dfrac{ab}{a+b}$ metres.

3. State and Prove Thales Theorem (or) Basic proportionality theorem.

4. State and Prove Angle Bisector Theorem.

5. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

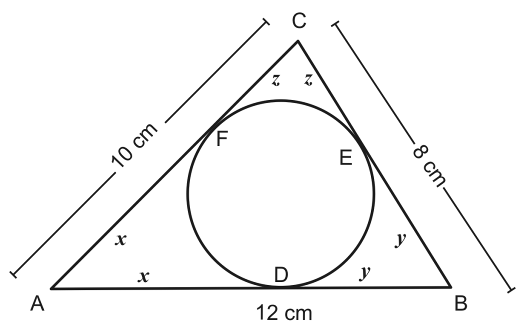
6. State and Prove Pythagoras Theorem (or) Baudhayana Theorem.

7. State and Prove Alternate Segment Theorem.

8. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of $20\ km/hr$ and the second train travels at $30\ km/hr$. After 2 hours, what is the distance between them?

9. In two concentric circles, a chord of length $16\ cm$ of larger circle becomes a tangent to the smaller circle whose radius is $6\ cm$. Find the radius of the larger circle.

10. A circle is inscribed in ΔABC having sides $8\ cm$, $10\ cm$ and $12\ cm$ as shown in figure, Find AD, BE and CF.



11. P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.

12. Show that in a triangle, the medians are concurrent.

13. A vertical stick of length $6\,\mathrm{cm}$ costs shadow $400\,\mathrm{cm}$ long on the ground and at the same time a tower casts a shadow $28\,\mathrm{m}$ long. Using similarity, find the height of the tower.

5. COORDINATE GEOMETRY

1. Find the area of the quadrilateral whose vertices are $(-9,-2)$, $(-8,-4)$, $(2, 2)$ and $(1,-3)$

Plot the points $(-9,-2)$, $(-8,-4)$, $(2, 2)$ and $(1,-3)$ and take them in counter-clockwise direction.

Let the vertices be $A(1,-3)$, $B(2, 2)$, $C(-9,-2)$ and $(-8,-4)$

Area of the quadrilateral ABCD

$$= \frac{1}{2}\begin{bmatrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & & 2 & & -9 & & -8 & & 1 \\ & \times & & \times & & \times & & \times & \\ -3 & & 2 & & -2 & & -4 & & -3 \end{vmatrix}$$

$$= \frac{1}{2}[\,(2-4+36+24)-(-6-18+16-4)]$$

$$= \frac{1}{2}[\,(62-4)-(16-28)]$$

$$= \frac{1}{2}[\,58-(\,-12\,)]$$

$$= \frac{1}{2}[\,58+12\,]$$

$$= \frac{1}{\cancel{2}} \times \overset{35}{\cancel{70}}$$

$$= 35 \text{ sq.units.}$$

2. Let $A(3,-4)$, $B(9,-4)$, $C(5,-7)$ and $D(7,-7)$ and show that ABCD is a trapezium.

Let the vertices of Trapezium are $A(3,-4)$, $B(9,-4)$, $C(5,-7)$ and $D(7,-7)$

$$A(3,-4) \qquad B(9,-4)$$
$$x_1 \; y_1 \qquad x_2 \; y_2$$

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-(-4)}{9-3}$$

$$= \frac{-4+4}{6}$$

$$= \frac{0}{6}$$

$$= 0 \qquad -(1)$$

$$C(5,-7) \qquad D(7,-7)$$
$$x_1 \; y_1 \qquad x_2 \; y_2$$

$$\text{Slope of } CD = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-7-(-7)}{7-5}$$

$$= \frac{-7+7}{2}$$

$$= \frac{0}{2}$$

$$= 0 \qquad -(2)$$

$$B(9,-4) \qquad C(5,-7)$$
$$x_1 \; y_1 \qquad x_2 \; y_2$$

$$\text{Slope of } BC = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7-(-4)}{5-9}$$
$$= \frac{-7+4}{-4}$$
$$= \frac{3}{4} \qquad -(3)$$

$$A(3,-4) \qquad D(7,-7)$$
$$x_1 \; y_1 \qquad x_2 \; y_2$$

$$\text{Slope of } AD = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7-(-4)}{7-3}$$
$$= \frac{-7+4}{4}$$
$$= \frac{-3}{4} \qquad -(4)$$

From (1) and (2), we see that slope of $\text{AB} = \text{slope of CD}$

$\Rightarrow$ AB and CD are parallel.

Also from (3) and (4) we see that, the lines BC and AD are not parallel, since their slope are not equal.

$\therefore$ The quadrilateral ABCD is a Trapezium.

Hence showed.

3. If the points $A(2, 2)$, $B(-2, -3)$, $C(1, -3)$ and $D(x, y)$ form a parallelogram, then find the value of x and y.

4. Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from $(6, 10)$ to $(14, 12)$ find the equation of the rod joining the buildings?

5. Find the equation of median of ΔABC through A where the vertices are $A(6, 2), B(-5, -1)$ and $C(1, 9)$.

6. Find the equation of a line passing through $(6, -2)$ and perpendicular to the line joining the points $(6, 7)$ and $(2, -3)$

7. Find the equation of the straight line intersection of lines $7x+3y=10,\ 5x-4y=1$ and parallel to the line $13x+5y+12=0$.

8. Find the equation of a straight line parallel to Y - axis and passing through the point of intersection of the lines $4x+5y=13$ and $x-8y+9=0$.

9. Find the equation of altitude of ΔABC through B where the vertices are $A(-3, 0),\ B(10, -2)$ and $C(12, 3)$.

10. A line makes positive intercepts on co-ordinate axes whose sum is 7 and its passes through $(-3,\ 8)$ Find its equation.

11. Show that the points $A(2.5, 3.5),\ B(10, -4),\ C(2.5, -2.5)$ and $D(-5, 5)$ form a parallelogram.

12. Without using Pythagoras theorem, show that the points $(1, -4),\ (2, -3)$ and $(4, -7)$ form a right angled triangle.

13. Using slope, show that the points $(-2, 5),\ (6, -1)$ and $(2, 2)$ are collinear.

14. If the points $P(-1, -4),\ Q(b, c)$ and $R(5, -1)$ are collinear and if $2b+c=4$ then find the values of b and c.

15. If the area of the quadrilateral formed by the vertices $A(-4, -2)$ $B(-3, k), C(3, -2)$ and $D(2, 3)$ taken in order is 28 sq.units, find the value of 'k'.

16. A quadrilateral has vertices $A(-4, -2),\ B(5, -1),\ C(6, 5)$ and $D(-7, 6)$ Show that the mid points of its sides form a parallelogram.

17. Find the equation of a line passing through $(1,\ 4)$ and perpendicular to the line joining the points $(2, 5)$ and $(4, 7)$.

18. The line joining the points $A(0, 5), B(4, 1)$ is a tangent to a circle whose centre C is at the point $(4,\ 4)$.

    (i) Find the equation of the line AB.

    (ii) Find the equation of the line through C, which is perpendicular to the line AB.

    (iii) Find the coordinates of the point of contact of line AB with the circle.

6. TRIGONOMETRY

1. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2 - 1}{p^2 + 1}$

<u>Given:</u> $\operatorname{cosec}\theta + \cot\theta = p$ ....(1)

We know that

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{p} \quad \text{...(2)}$$

$$(1) + (2) \Rightarrow \quad 2\operatorname{cosec}\theta = p + \frac{1}{p}$$

$$2\operatorname{cosec}\theta = \frac{p^2 + 1}{p} \quad \text{...(3)}$$

$$(1) - (2) \Rightarrow \quad 2\cot\theta = p - \frac{1}{p}$$

$$2\cot\theta = \frac{p^2 - 1}{p} \quad \text{...(4)}$$

$$\frac{(4)}{(3)} \Rightarrow \quad \frac{\cancel{2}\cot\theta}{\cancel{2}\operatorname{cosec}\theta} = \frac{(p^2 - 1)/p}{(p^2 + 1)/p}$$

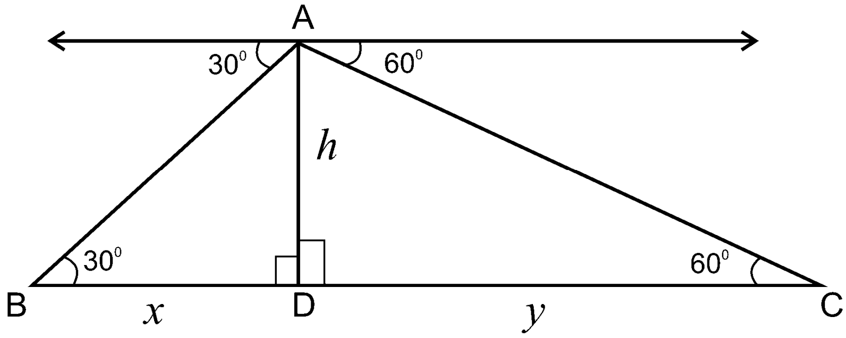
$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \frac{p^2 - 1}{p} \times \frac{p}{p^2 + 1}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{1} = \frac{p^2 - 1}{p^2 + 1}$$

$$\therefore \cos\theta = \frac{p^2 - 1}{p^2 + 1}$$

Hence Proved

2. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is 'h' meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.



<u>Given</u>

A → top of the light house

AD → height of the light house

B and C → two ships

Let BD = x and CD = y

$$\tan\theta = \frac{Opp.Side}{Adj.Side}$$

$In\ \Delta\, ADB,$

$$\tan 30° = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}h$$

$In\ \Delta\, ADC,$

$$\tan 60° = \frac{AD}{DC}$$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}}$$

Distance between two ships = BD+CD

$= x + y$

$= \sqrt{3}h + \frac{h}{\sqrt{3}}$

$= \frac{3h + h}{\sqrt{3}}$

$= \frac{4h}{\sqrt{3}}$

Hence showed.

3. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are $45°$ and $60°$ respectively. Find the height of the tower.

4. Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is $40°$. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m, calculate

   (i) the distance between the point X and the top of the smaller tree.
   (ii) the horizontal distance between the two trees. $(\cos 40° = 0.7660)$

5. To a man standing outside his house, the angles of elevation of the top and bottom of a window are $60°$ and $45°$ respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window?

6. From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be $30°$ and $45°$ respectively. Find the height of the tree.

7. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are $60°$ and $30°$ respectively. Find the distance between the two boats.

8. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are $45°$ and $30°$ respectively. Find the height of the second tree.

9. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be $38°$ and $60°$ respectively. Find the height of the lamp post. $(\tan 38° = 0.7813, \sqrt{3} = 1.732)$

10. From the top of a 12 m high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $30°$. Determine the height of the tower.

11. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are $60°$ and $30°$ respectively. Find

    (i) The height of the lamp post.
    (ii) The difference between height of the lamp post and the apartment.
    (iii) The distance between the lamp post and the apartment.

12. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are $30°$ and $45°$ respectively. If the lighthouse is 200 m high, find the distance between the two ships. $(\sqrt{3} = 1.732)$

13. As observed from the top of a 60 m high light house from the sea level, the angles of depression of two ships are $28°$ and $45°$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. $(\tan 28° = 0.5317)$

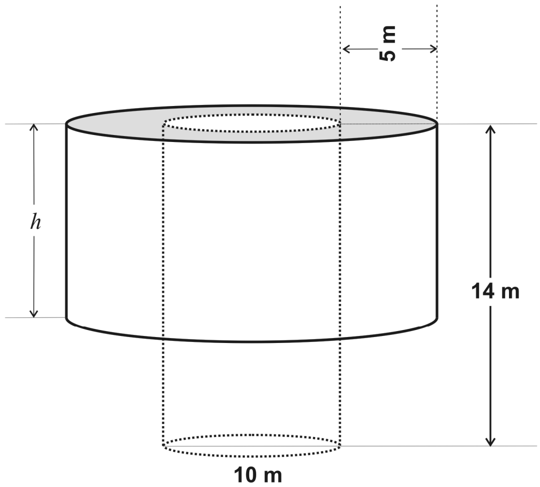
14. If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $h\dfrac{(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 - \tan\theta_1}$.

7. MENSURATION

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.

Solution:

$$Well: \quad diameter = 10m, \quad radius = 5m, \quad depth = 14m$$



$$\begin{aligned} \text{Volume } (V_1) &= \pi r^2 h \\ &= \pi \times 5^2 \times 14 \\ &= \pi \times 5 \times 5 \times 14 \end{aligned}$$

Embankment: width = 5m, Inner radius (r) = 5m,

Outer radius = $= r + w = 5 + 5 = 10$ m

Let the height be 'h' m

$$\begin{aligned} \text{Volume } (V_2) &= \pi h(R^2 - r^2) \\ &= \pi \times h \times (10^2 - 5^2) \\ &= \pi \times h \times (100 - 25) \\ &= \pi \times h \times 75 \end{aligned}$$

Here $V_2 = V_1$ (Vol.of earth taken out=Vol.of earth spread all around)

$$\pi \times h \times 75 = \pi \times 5 \times 14 \times 5$$

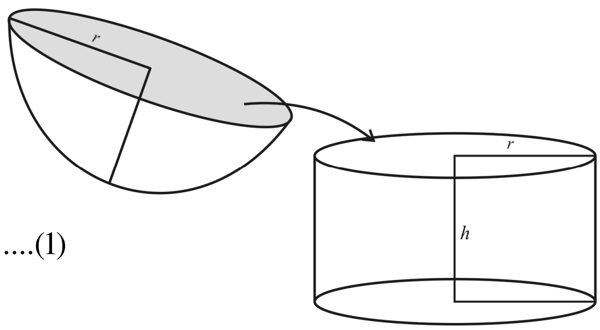
$$h = \frac{\not{\pi} \times \not{5} \times 14 \times \not{5}}{\not{\pi} \times \not{75}_3}$$

$$= 4.67$$

Height of the embankment $= 4.67$ m.

2. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution:



Hemispherical bowl:

Radius = r,

Volume $(V_1) = \frac{2}{3}\pi r^3$ ....(1)

Cylindrical vessel:

Radius = r,

height = h

Radius is 50% more than its height.

$$r = h + \frac{1}{2}h$$

$$= 1\frac{1}{2}h$$

$$= \frac{3}{2}h$$

$$h = \frac{2}{3}r$$

$$\text{Volume } (V_2) = \pi r^2 h$$

$$= \pi \times r^2 \times \frac{2}{3}r$$

$$= \frac{2}{3}\pi r^3 \quad .....(2)$$

From (1) and (2), we see that $V_1 = V_2$

100% of the juice that can be transferred from the bowl into the cylindrical vessel.

3. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.

4. The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent, if the width of the rectangular canvas is 4 m?

5. The radius and height of a cylinder are in the ratio $5:7$ and its curved surface area is 5500 sq.cm. Find its radius and height.

6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 sq.cm, how many caps can be made with radius 5 cm and height 12 cm?

7. A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

8. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.

9. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?

10. A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

11. A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

12. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

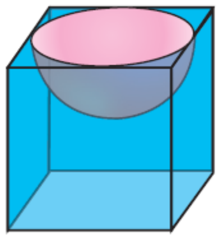
13. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.

14. An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

15. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

16. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

17. A hemispherical section is cut out from one face of a cubical block such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.



18. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

8. STATISTICS AND PROBABILITY

1. The Marks scored by 10 students in a class test are $25,\ 29,\ 30,\ 33,\ 35,\ 37,\ 38,\ 40,\ 44,\ 48$. Find the standard deviation.

Mean $\overline{x} = \dfrac{\sum x}{n}$

$$\text{Mean} = \frac{25+29+30+33+35+37+38+40+44+48}{10} = \frac{359}{10} = 35.9$$

Assumed Mean $A = 36$ (Since mean value is not an integer)

x	$d = x - 36$	d^2
25	-11	121
29	-7	49
30	-6	36
33	-3	9
35	-1	1
37	1	1
38	2	4
40	4	16
44	8	64
48	12	144
	$\sum d = -1$	$\sum d^2 = 445$

$$\text{Standard Deviation } \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{445}{10} - \left(\frac{-1}{10}\right)^2}$$

$$= \sqrt{44.5 - \left(\frac{1}{100}\right)}$$

$$= \sqrt{44.5 - 0.01}$$

$$= \sqrt{44.49}$$

$$\simeq 6.67$$

$\therefore$ The standard deviation of the given data is 6.67

2. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$
$$n(S) = 8$$

$$A \rightarrow \text{getting exactly two heads}$$
$$A = \{HHT, HTH, THH\}$$
$$n(A) = 3$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

$$B \rightarrow \text{getting atleast one tail}$$
$$B = \{HHT, HTH, THH, TTH, THT, HTT, TTT\}$$
$$n(B) = 7$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

$$C \rightarrow \text{getting two consecutive heads}$$
$$C = \{HHH, HHT, THH\}$$
$$n(C) = 3$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$
$$A \cap B = \{HHT, HTH, THH\}$$
$$n(A \cap B) = 3$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$$
$$B \cap C = \{HHT, THH\}$$
$$n(B \cap C) = 2$$
$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{2}{8}$$
$$A \cap C = \{HHT, THH\}$$
$$n(A \cap C) = 2$$
$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{8}$$

$\therefore$ Probability of getting exactly two heads or atleast one tail or two consecutive heads is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
$$= \frac{3}{8} + \frac{7}{8} + \cancel{\frac{3}{8}}, - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}} = \frac{3+7-2}{8} = \frac{10-2}{8} = \frac{8}{8} = 1$$

3. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only $32, 35, 37, 30, 33, 36, 35$ and 37 pages. Find the standard deviation of the pages yet to be completed by them.

4. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

5. The marks scored by the students in a slip test are given below

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

6. Find the coefficient of variation of $24, 26, 33, 37, 29, 31$.

7. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

8. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is $4:3$. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

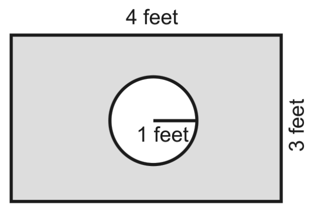
9. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

10. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

11. Find the mean and variance of the first n natural numbers.

12. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?

13. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



14. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(i) white (ii) black or red (iii) not white

15. If $n = 5$, $\overline{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

2 MARK QUESTIONS

1. RELATIONS AND FUNCTIONS

1. If $A\times B=\{(3,2),(3,4),(5,2),(5,4)\}$ then find A and B.

Solution $A\times B=\{(3,2),(3,4),(5,2),(5,4)\}$

$A=\{Set\ of\ all\ first\ elements\ of\ A\times B\} = \{3,5\}$

$B=\{Set\ of\ all\ second\ elements\ of\ A\times B\} = \{2,4\}$

2. Let $A=\{1,2,3\}$, and $B=\{x/x\ is\ a\ prime\ numbers\ less\ than\ 10\}$ Find $A\times B$ and $B\times A$

Given

$A = \{1,2,3\}$

$B = \{x/x\ is\ a\ prime\ numbers\ less\ than\ 10\}$

$= \{2,3,5,7\}$

$A\times B = \{1,2,3\}\times\{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7)\}$

$B\times A = \{2,3,5,7\}\times\{1,2,3\}$

$= \{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),(5,2),(5,3),(7,1),(7,2),(7,3)\}$

3. A relation $'f'$ is defined by $f(x)=x^2-2$ where $x\in\{-2,-1,0,1,3\}$;
(i) List the elements of f (ii) Is $'f'$ a function

Given: $f(x) = x^2-2$

$f(-2) = (-2)^2-2 = 4-2 = 2$

$f(-1) = (-1)^2-2 = 1-2 = -1$

$f(0) = (0)^2-2 = 0-2 = -2$

$f(1) = (1)^2-2 = 1-2 = -1$

$f(3) = (3)^2-2 = 9-2 = 7$

(i) $f=\{(-2,2),(-1,-1),(0,-2),(1,-1),(3,7)\}$

(ii) Yes, It is a function since each element in the domain of $'f'$ has an unique image.

4. A function $'f'$ is defined by $f(x)=x^2-5x+6$ find $f(2a)$

Given: $f(x) \quad = x^2-5x+6$

$$f(2a)=(2a)^2-5(2a)+6$$
$$=4a^2-10a+6$$

5. A function $'f'$ is defined by $f(x)=3-2x$ find x such that $f(x^2)=[f(x)]^2$

Given: $f(x)=3-2x$

$$f(x^2) \quad =3-2x^2$$
$$[f(x)]^2=(3-2x)^2$$
$$=(3)^2-2(3)(2x)+(2x)^2 \qquad [\because (a-b)^2=a^2-2ab+b^2]$$
$$=9-12x+4x^2$$

It is given that $f(x^2) \quad =[f(x)]^2$

$$3-2x^2 \quad =9-12x+4x^2$$
$$9-12x+4x^2 \quad =3-2x^2$$
$$9-12x+4x^2-3+2x^2=0$$
$$6x^2-12x+6=0$$

$(\div)$ by 6, on both sides, we have $x^2-2x+1=0$

$$(x-1)(x-1)=0$$

$(x-1)=0$ | $(x-1)=0$
$x=1$ | $x=1$

$$\therefore x=1,1$$

6. Let $A=\{1,2,3,4\}$ and $B=\{N\}$, Let $f:A\to B$ be a function defined by $f(x)=x^3$, then find the range of f

Given: $f(x)=x^3$

$$f(1) \quad =1^3 \quad = \quad 1$$
$$f(2) \quad =2^3 \quad = \quad 8$$
$$f(3) \quad =3^3 \quad =27$$
$$f(4) \quad =4^3 \quad =64$$

$\therefore$ The range of f $=\{1,8,27,64\}$

7. Show that the function $f: N \to N$ defined by $f(x) = 2x - 1$ is one-one but not onto.

Given: $f(x) = 2x - 1$

$$f(1) = 2(1) - 1 = 2 - 1 = 1$$
$$f(2) = 2(2) - 1 = 4 - 1 = 3$$
$$f(3) = 2(3) - 1 = 6 - 1 = 5$$
$$f(4) = 2(4) - 1 = 8 - 1 = 7$$

Here, Every element in the domain has distinct images in the co-domain.

$\therefore\ f: N \to N$ is a one-one function.

A function $f: N \to N$ is said to be onto function, if the range is equal to the co-domain of $'f'$ Here the range of $'f'$ is not equal to the co-domain. Therefore it is one-one but onto function.

8. Find k, if $fof(k) = 5,$ where $f(k) = 2k - 1$

Given: $fof(k) = 5$

$$f[f(k)] = 5$$
$$f(2k-1) = 5$$
$$2(2k-1) - 1 = 5$$
$$4k - 2 - 1 = 5$$
$$4k - 3 = 5$$
$$4k = 5 + 3$$
$$4k = 8$$
$$k = \frac{8}{4}$$
$$k = 2$$

9. If $f(x) = x - 6,\ g(x) = x^2,$ find fog

Given: $f(x) = x - 6$

$$g(x) = x^2$$
$$fog = f[g(x)]$$
$$= f(x^2)$$
$$= x^2 - 6$$

10. Find the value of k such that $fog = gof$, $f(x) = 3x+2$, $g(x) = 6x-k$

Given: $f(x) = 3x+2$

$g(x) = 6x-k$

$fog = f[g(x)]$ $= f(6x-k)$ $= 3(6x-k)+2$ $= 18x-3k+2$	$gof = g[f(x)]$ $= g(3x+2)$ $= 6(3x+2)-k$ $= 18x+12-k$

It is given that $fog = gof$

$$\Rightarrow 18x-3k+2 = 18x+12-k$$
$$-3k+k = 12-2$$
$$-2k = 10$$
$$k = \frac{-10}{2}$$
$$k = -5$$

11. If $B \times A = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$ then find A and B.

12. Let $A = \{3,4,7,8\}$ and $B = \{1,7,10\}$ is $R = \{(3,1),(4,12)\}$ a relation from A to B?

13. A relation R is given by the set $\{(x,y) / y = x+3,\ x \in (0,1,2,3,4,5)\}$. Determine its domain and range.

14. Let $A = \{1,2,3\}, B = \{4,5,6,7\}$, and $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that 'f' is one-one but not onto function.

15. Define a function.

16. Represent the function $f = \{(1,2),\ (2,2),\ (3,2),\ (4,3),\ (5,4)\}$ through

    i) an arrow diagram. (ii) a table form

17. Let $f(x) = x^2 - 1$ find $fofof$

18. Define Reciprocal function.

19. If $A = \{-2,-1, 0,1,2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$, then find B.

20. Let $f = \{(-1,3),(0,-1),(2,-9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.

2. NUMBERS AND SEQUENCES

1. Using Euclid's Division Algorithm to find the HCF of 340 and 412.

Given : $a = 412, \quad b = 340$ (Here $a > b$)

Using EDA, we get [Euclid's division Algorithm, $a = bq + r$]

$$\begin{aligned}
412 &= 340\times1+72\\
340 &= 72\times4+52\\
72 &= 52\times1+20\\
52 &= 20\times2+12\\
20 &= 12\times1+8\\
12 &= 8\times1+4\\
8 &= 4\times2+0
\end{aligned}$$

The remainder $=0$, the divisor at this stage $=4$

$\therefore$ HCF of 340 and 412 is 4

2. Compute the value of x, if $10^4 \equiv x(\text{mod}\,19)$

$$\begin{aligned}
10^2 = 100 &\equiv 5(\text{mod}\,19)\\
10^4 = (10^2)^2 &\equiv 5^2(\text{mod}\,19)\\
&\equiv 25(\text{mod}\,19)\\
x(\text{mod}\,19) &\equiv 6(\text{mod}\,19)
\end{aligned}$$

$$\therefore x = 6$$

3. Solve $5x \equiv 4(\text{mod}\,6)$

$$5x \equiv 4(\text{mod}\,6)$$

$$\Rightarrow 5x - 4 = 6k \quad \textit{for some integer } k$$

$$5x = 6k + 4$$

$$x = \frac{6k+4}{5}$$

$$\text{when } k = 1,\ x = \frac{6(1)+4}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$$

$$k = 6,\ x = \frac{6(6)+4}{5} = \frac{36+4}{5} = \frac{40}{5} = 8$$

$$k = 11,\ x = \frac{6(11)+4}{5} = \frac{66+4}{5} = \frac{70}{5} = 14$$

$$x = 2,\ 8,\ 14, \ldots$$

4. The ratio of 6th and 8th term of an A.P. is 7:9. Find the ratio of 9th term to 13th term.

Given $\frac{t_6}{t_8} = \frac{7}{9}$

$$\Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9}$$

$$9(a+5d) = 7(a+7d)$$

$$9a+45d = 7a+49d$$

$$9a-7a = 49d-45d$$

$$2a = 4d$$

$$a = \frac{4d}{2}$$

$$a = 2d$$

$$\frac{t_9}{t_{13}} = \frac{a+8d}{a+12d}$$

$$= \frac{2d+8d}{2d+12d}$$

$$= \frac{10d}{14d} = \frac{5}{7}$$

$$\therefore t_9 : t_{13} = 5:7$$

5. If $3+k,\ 18-k,\ 5k+1$ are in A.P., then find k.

Given: $3+k,\ 18-k,\ 5k+1$ *are in A.P.*

$$2(18-k) = 3+k+5k+1$$

$$36-2k = 6k+4$$

$$6k+4 = 36-2k$$

$$6k+2k = 36-4$$

$$8k = 32$$

$$k = \frac{32}{8} = 4$$

$$k = 4$$

6. Find the 8^{th} term of an G.P. $9, 3, 1...$

Given: $a = 9,\quad r = \frac{3}{9} = \frac{1}{3},\quad n = 8$

$$t_n = ar^{n-1}$$

$$t_8 = 9\left(\frac{1}{3}\right)^{8-1}$$

$$= 9\left(\frac{1}{3}\right)^{7}$$

$$= 3\times 3\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$$

$$= \frac{1}{243}$$

7. Find the sum to infinity of $9+3+1+............$

Given $a = 9,\quad r = \frac{3}{9} = \frac{1}{3} < 1$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \left(\frac{9}{1-\frac{1}{3}}\right)$$

$$= \left(\frac{9}{2/3}\right)$$

$$= 9\times\frac{3}{2}$$

$$= \frac{27}{2}$$

8. Find the sum of the series $1+4+9+16+\ ....\ +225$

$$1+4+9+16+\ ....\ +225$$

$$=1^2+2^2+3^2+\ \ldots\ldots\ +15^2$$
$$=\sum 15^2$$
$$=\left[\frac{n(n+1)(2n+1)}{2}\right]_{n=15}$$
$$=\frac{\overset{5}{\cancel{15}}\times\overset{8}{\cancel{16}}\times 31}{\cancel{6}} = 40\times 31$$
$$=1240$$

9. Find x, so that $x+6,\ \ x+12$ and $x+15$ are consecutive terms of G.P.

<u>Given</u> $x+6,\ \ x+12,\ \ x+15$ are three consecutive terms of a G.P.

$$(x+12)^2 = (x+6)(x+15)$$
$$x^2+24x+144 = x^2+6x+15x+90$$
$$24x+144 = 21x+90$$
$$24x-21x = 90-144$$
$$3x = -54$$
$$x = \frac{\overset{18}{\cancel{-54}}}{\cancel{3}}$$
$$\therefore x = -18$$

10. Find all the positive integers, when divided by 3 leaves remainder 2.

11. If the HCF of 210 and 55 is expressible in the form $55x-325$, find x.

12. If 'a' and 'b' are two positive integers such that $a^b\times b^a=800$. Find 'a' and 'b'.

13. If $13824 = 2^a\times 3^b$ then find 'a' and 'b'

14. Find the HCF of 252525 and 363636.

15. What is the time 100 hours after 7 a.m.?.

16. What is the rational form of the number $0.\overline{123}$

17. Find the G.P. in which the 2nd term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$

18. Find the least number that is divisible by the first ten natural numbers.

19. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

20. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is $0°\,C$ and the sum of the temperatures from Wednesday to Friday is $18°\,C$. Find the temperature on each of the five days.

3. ALGEBRA

1. Find the L.C.M of $x^2 - 27,\ (x-3)^2,\ x^2 - 9$

$$\begin{aligned} x^3 - 27 &= x^3 - 3^3 = (x-3)(x^3+3x+9) \\ (x-3)^2 &= (x-3)^2 \\ x^2 - 9 &= (x-3)(x+3) \\ \therefore LCM &= (x-3)^2(x+3)(x^2+3x+9) \end{aligned}$$

2. Simplify: $\dfrac{5t^2}{4t-8} \times \dfrac{6t-12}{10t}$

$$= \frac{5t^2}{4t-8} \times \frac{6t-12}{10t}$$

$$= \frac{\cancel{5}t^{\cancel{2}}}{\cancel{4}\,\cancel{(t-2)}} \times \frac{\cancel{6}\,\cancel{(t-2)}}{\cancel{10t}}$$

$$= \frac{3t}{2}$$

3. Solve $2x^2 - 2\sqrt{6}x + 3 = 0$

Given $\quad 2x^2 - 2\sqrt{6}x + 3 = 0$

$$(2x - \sqrt{6})(2x - \sqrt{6}) = 0$$

$$\begin{aligned} (2x - \sqrt{6}) &= 0 \\ 2x &= \sqrt{6} \\ x &= \frac{\sqrt{6}}{2} = \frac{\sqrt{3}\times\cancel{\sqrt{2}}}{\sqrt{2}\times\cancel{\sqrt{2}}} \\ &= \frac{\sqrt{3}}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} (2x - \sqrt{6}) &= 0 \\ 2x &= \sqrt{6} \\ x &= \frac{\sqrt{6}}{2} \\ &= \frac{\sqrt{3}}{\sqrt{2}} \end{aligned}$$

Rough work

P	S
6	$-2\sqrt{6}$

$\dfrac{-\sqrt{6}}{2} \quad \dfrac{-\sqrt{6}}{2}$

Solution set $= \left\{ \dfrac{\sqrt{3}}{\sqrt{2}}, \dfrac{\sqrt{3}}{\sqrt{2}} \right\}$

4. Solve $2x^2 - 5x + 2 = 0$ by formula method.

Given

$a = 2,$

$b = -5,$

$c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$$= \frac{5+3}{4} \text{ (or) } \frac{5-3}{4}$$

$$= \frac{\not{8}}{\not{4}} \text{ (or) } \frac{\not{2}}{\not{4}}$$

$$= 2 \quad \text{(or)} \quad \frac{1}{2}$$

Solution set $= \left\{2, \ \frac{1}{2}\right\}$

5. If α and β are the roots of $3x^2+7x-2=0$, find the values of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Let α and β are the roots of $3x^2+7x-2=0$

$$SOR=\alpha+\beta=\frac{-b}{a}=-\frac{7}{3}$$

$$POR=\alpha\beta=\frac{c}{a}=-\frac{2}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(\frac{-7}{3}\right)^2-2\left(\frac{-2}{3}\right)}{\left(\frac{-2}{3}\right)}$$

$$=\frac{\left(\frac{49}{9}\right)+\left(\frac{4}{3}\right)}{\left(\frac{-2}{3}\right)}$$

$$=\frac{\left(\frac{49+12}{9}\right)}{\left(\frac{-2}{3}\right)}$$

$$=\frac{\left(\frac{61}{9}\right)}{\left(\frac{-2}{3}\right)}$$

$$=\frac{61}{\cancel{9}_{3}}\times\frac{-\cancel{3}}{2}$$

$$=-\frac{61}{6}$$

6. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$, Find AB

Given $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$

$$B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$$

$$AB = \overrightarrow{\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}} \downarrow \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} (1 \ \ 2 \ \ 0)\begin{pmatrix} 8 \\ 2 \\ 5 \end{pmatrix} & (1 \ \ 2 \ \ 0)\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} & (1 \ \ 2 \ \ 0)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ (3 \ \ 1 \ \ 5)\begin{pmatrix} 8 \\ 2 \\ 5 \end{pmatrix} & (3 \ \ 1 \ \ 5)\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} & (3 \ \ 1 \ \ 5)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$$

7. A ball rolls down a slope and travels a distance $d = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.

8. Determine the nature of roots of $x^2 - x - 1 = 0$.

9. Reduce the rational expression to its lowest form $\dfrac{x^2 - 16}{x^2 + 8x + 16}$

10. Pari needs 4 hours to complete the work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?

11. Find the square root of $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

12. Find the zeros of quadratic expression $x^2 + 8x + 12$

13. Solve $\sqrt{a(a-7)} = 3\sqrt{2}$

14. Solve $\frac{5x+7}{x-1} = 3x + 2$ by completing the square method.

15. The number of volleyball games that must be scheduled in a league with n teams is given by $G(n) = \frac{n^2 - n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

16. Solve $2x^2 - x - 1 = 0$ by formula method.

17. Find the value of k for which the roots of quadratic equation $(5k-6)x^2 + 2kx + 1 = 0$ are real and equal.

18. Write $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$ in terms of $\alpha + \beta$ and $\alpha\beta$

19. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find $3\text{A} - 9\text{B}$.

20. Show that the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ satisfy commutative property under matrix multiplication.

21. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then, verify $(A^T)^T = A$

22. If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$, then find $2A + B$

23. Construct a 3 x 3 matrix, whose elements are given by $a_{ij} = |i - 2j|$

24. If $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$ verify $A^2 = I$

25. $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then, prove that $AA^T = I$

26. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then, find the transpose of $-A$

4. GEOMETRY

1. The perimeter of two similar triangle ABC and PQR are respectively 36 cm and 24 cm. If $PQ = 10$ cm, find AB.

Given: Perimeter of ΔABC = 36 *cm*

Perimeter of ΔPQR = 24 *cm*

PQ = 10 *cm*

We know that the ratio of the corresponding sides of similar triangles is equal to the ratio of perimeter.

In ΔABC and ΔPQR

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

$$\frac{AB}{10} = \frac{36}{24}$$

$$AB = \frac{{}^{3}\cancel{36} \times \cancel{10}^{5}}{\cancel{24}_{2}}$$

$$= 15 \text{ cm}$$

$$\therefore \text{ AB } = 15 \text{ cm}$$

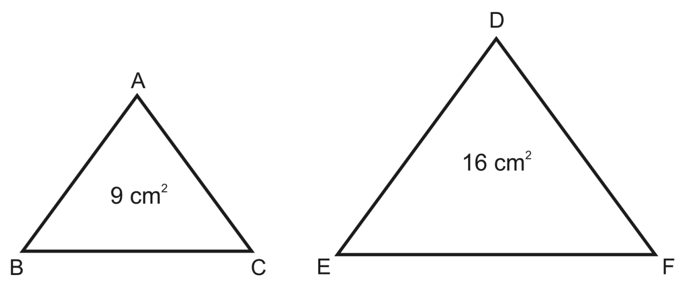
2. If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is 9 cm^2 and the area of ΔDEF is 16 cm^2 and $BC = 2.1 \; cm$. Find the length of EF.

Given Area of $\Delta ABC = 9 \text{ cm}^2$

Area of $\Delta DEF = 16 \text{ cm}^2$

$BC = 2.1$ cm

$EF = ?$



We know that the area of two similar triangle is equal to the ratio of squares of any two corresponding sides.

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{(BC)^2}{(EF)^2}$$

$$\frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$(EF)^2 = \frac{(2.1)^2 \times 16}{9}$$

$$EF^2 = \frac{(2.1)^2 \times 4^2}{3^2}$$

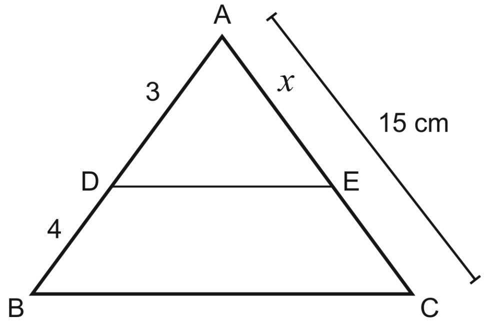
$$EF = \frac{2.1 \times 4}{3}$$

$$= \frac{8.4}{3}$$

$$= 2.8\,\text{cm}$$

$\therefore$ Length of EF = 2.8 cm.

3. In $\Delta ABC, D$ and E are points on the sides of AB and AC respectively such that $DE \parallel BC$ if $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15\,\text{cm}$. Find AE.



Given:

$$\frac{AD}{DB} = \frac{3}{4}$$

$$AC = 15\,\text{cm}$$

By the corollary of Thales theorem,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{3}{7} = \frac{x}{15}$$

$$7x = 45$$

$$x = \frac{45}{7}$$

$$= 6.43$$

$$\Rightarrow AE = 6.43\,\text{cm}$$

4. In ΔABC, AD is the bisector of $\angle A$ meeting side BC at D, if $AB = 10\,\text{cm}$, $AC = 14\,\text{cm}$ and $BC = 6\,\text{cm}$, find BD and DC.

Given

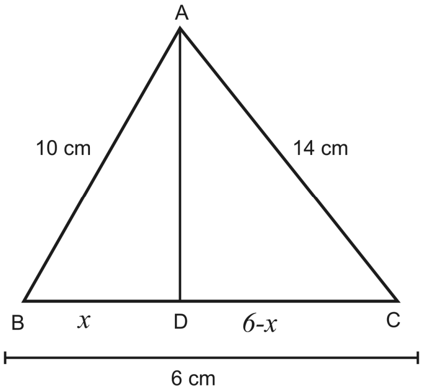
AB = 10cm

AC = 14 cm

BC = 6 cm

BD = ?

DC = ?



By Angle Bisector theorem,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{14} = \frac{x}{6-x}$$

$$14x = 10(6-x)$$
$$14x = 60-10x$$

$$14x+10x = 60$$

$$24x = 60$$

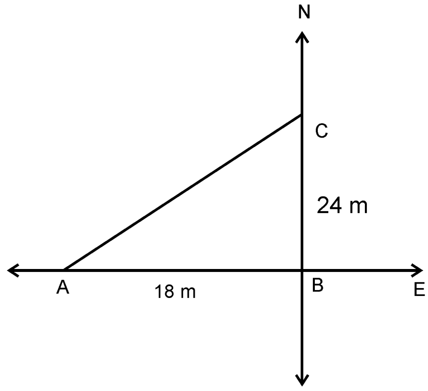
$$x = \frac{60}{24}$$

$$x = 2.5$$

$$\Rightarrow BD = 2.5$$
$$DC = 6-x$$
$$= 6-2.5$$
$$= 3.5$$

$$\boxed{\therefore BD = 2.5\,\text{cm} \quad DC = 3.5\,\text{cm}}$$

5. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point.



Starting point $\rightarrow A$

Current position $\rightarrow AC$

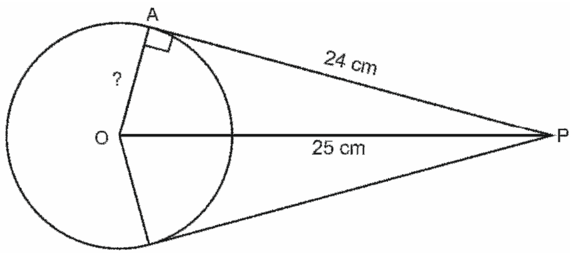
By Pythagoras theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{18^2 + 24^2}$$
$$= \sqrt{324 + 576}$$
$$= \sqrt{900}$$
$$= 30$$

$\therefore$ The distance from the position of the starting point is 30 m.

6. The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

In the right angled ΔOAP, By Pythagoras theorem,



$$OA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{25^2 - 24^2}$$
$$= \sqrt{625 - 576}$$
$$= \sqrt{49}$$
$$= 7$$

$\therefore$ radius of the circle $r = 7$ cm.

7. A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.

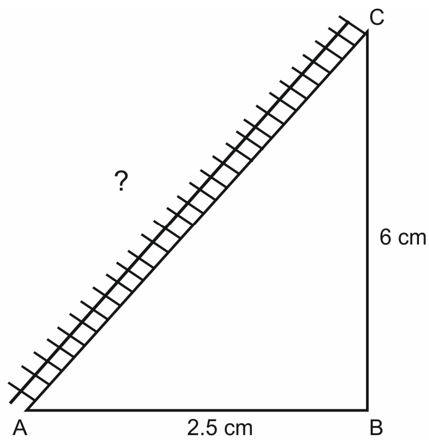
Given $AB = 2.5$ m

$BC = 6$ m

AC $\rightarrow$ ladder

BC $\rightarrow$ wall with window at C

Length of ladder $AC = ?$



In the right angled ΔABC

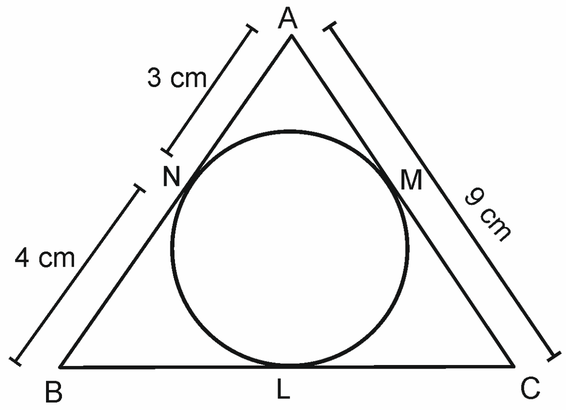
By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{(2.5)^2 + 6^2}$$
$$= \sqrt{6.25 + 36}$$
$$= \sqrt{42.25}$$
$$= 6.5$$

$\therefore$ the length of the ladder is $6.5\,\text{m}$

8. If ΔABC is similar to ΔDEF such that $BC = 3$ cm, $EF = 4$ cm, and area of $\Delta ABC = 54\,\text{cm}^2$. Find the area of ΔDEF.
9. If the radii of two circles are 4 cm and 5 cm then find the chord of one circle which is a tangent to the other circle.
10. Find the length of tangent drawn from a point whose distance from the centre of circle is 5 cm and radius of the circle is 3 cm.
11. The hypotenuse of a triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the side of the triangle.
12. What length of ladder is needed to reach a height of 7 feet along the wall when the base of the ladder is 4 feet from the wall?
13. AD is bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC.
14. In figure, ΔABC is circumscribing a circle. Find the length of BC.



5. COORDINATE GEOMETRY

1. Find the area of the triangle whose vertices are $(1,-1),(-4,6)$ and $(-3,-5)$

Let the vertices be $A(1,-1)$, $B(-4,6)$ and $C(-3,-5)$

Area of the triangle ABC

$$= \frac{1}{2}\left| \begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array} \right|$$

$$= \frac{1}{2}\left| \begin{array}{cccc} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{array} \right|$$

$$= \frac{1}{2}[\,(6+20+3)-(4-18-5)]$$

$$= \frac{1}{2}[\,29-(4-23)]$$

$$= \frac{1}{2}[\,29-(-19\,)]$$

$$= \frac{1}{2}[\,29+19\,]$$

$$= \frac{1}{\cancel{2}} \times \overset{24}{\cancel{48}}$$

$= 24$ sq.units.

2. Find the equation of straight line passing through the points $(2, 3)$ and $(-7, -1)$.

$A(2,\ 3)$ $B(-7,-1)$

$x_1\ y_1$ $x_2\ y_2$

Equation of straight line passing through two points is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$$

$$\frac{y-3}{-4} = \frac{x-2}{-9}$$

$$-9(y-3) = -4(x-2)$$

$$-9y+27 = -4x+8$$

$$4x-9y+27-8 = 0$$

$$\Rightarrow 4x-9y+19 = 0$$

3. Find the intercepts made by the line $3x-2y-6=0$ on the coordinate axes.

Given: Equation of line is $3x-2y-6=0$

$$3x-2y-6 = 0$$

$$3x-2y = 6$$

Divide by 6, on both sides

$$\frac{3x}{6} - \frac{2y}{6} = \frac{6}{6}$$

$$\frac{x}{2} - \frac{y}{3} = 1$$

$$\frac{x}{2} + \frac{y}{-3} = 1$$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$

$x-$intercept $a=2$

$y-$intercept $b=-3$

4. Show that the straight lines $x-2y-3=0$ and $6x+3y+8=0$ are perpendicular.

$x-2y-3=0$

Slope $m_1 = \frac{-coefficient\ of\ x}{coefficient\ of\ y}$

$= \frac{-1}{-2}$

$= \frac{1}{2}$

$6x+3y+8=0$

Slope $m_2 = \frac{-coefficient\ of\ x}{coefficient\ of\ y}$

$= \frac{-6^{2}}{3}$

$= -2$

$$m_1 \times m_2 = \frac{1}{2} \times (-2^{1})$$
$$= -1$$

$\Rightarrow$ The two lines are perpendicular

Hence showed

5. Show that the points $(a,\ b+c)$, $(b,\ c+a)$ and $(c,\ a+b)$ are collinear.
6. Find the slope of line joining two points $(-6,1)$ and $(-3,2)$.
7. Find the equation of straight line whose inclination is 45^0 and y intercept is 11.
8. Find the equation of straight line passing through the points $(3,\ -4)$ and having slope $\frac{-5}{7}$.
9. Find the equation of line whose intercepts on the x and y axes are 4 and -6.
10. Find the slope of the straight line $7x - \frac{3}{17} = 0$.
11. Show that the straight lines $2x+3y-8=0$ and $4x+6y+18=0$ are parallel.
12. Find the slope of the line which is parallel to $3x-7y=11$.

6.TRIGONOMETRY

1. Prove that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

$$L.H.S \quad \tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1)$$
$$= (\sec^2\theta - 1)(\sec^2\theta)$$
$$= \sec^4\theta - \sec^2\theta$$
$$= R.H.S$$

Hence Proved

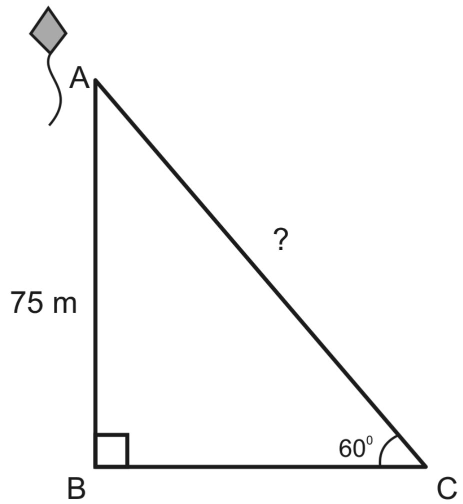
2. Prove that $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$

$$L.H.S : \sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B$$
$$= \sin^2 A\cos^2 B + \sin^2 A\sin^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B$$
$$= \sin^2 A(\cos^2 B + \sin^2 B) + \cos^2 A(\sin^2 B + \cos^2 B)$$
$$= \sin^2 A(1) + \cos^2 A(1)$$
$$= \sin^2 A + \cos^2 A$$
$$= 1$$
$$= R.H.S$$

Hence Proved

3. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60°$. Find the length of the string, assuming that there is no slack in the string.

Given: Height of the kite AB = 75 m   Length of the string AC = ?



In the right angled ΔABC,

$$\sin\theta = \frac{AB}{AC}$$

$$\sin 60° = \frac{75}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$

$$AC = \frac{2\times 75}{\sqrt{3}}$$

$$AC = \frac{150}{\sqrt{3}}$$

$$= \frac{150}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\overset{50}{\cancel{150}}\sqrt{3}}{\cancel{3}}$$

$$= 50\sqrt{3}\ m$$

4. Prove that $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

5. Prove that $\dfrac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

6. Prove that $\dfrac{\sec\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta} = \cot\theta$

7. Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.

8. Find the angle of elevation of the top of a tower from a point on the ground, which is $30\,m$ away from the foot of a tower of height $10\sqrt{3}m$.

9. A tower stands vertically on the ground. From a point on the ground, which is $48\,m$ away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

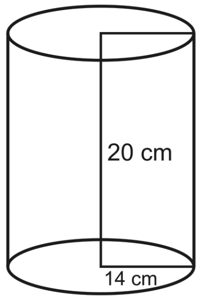
10. From the top of a rock $50\sqrt{3}m$ high, the angle of depression of a car on the ground is observed to be 30^0. Find the distance of the car from the rock.

11. A player sitting on the top of a tower of height $20\,m$ observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. $(\sqrt{3} = 1.732)$

7. MENSURATION

1. A cylindrical drum has a height of 20cm and base radius 14cm. Find its T.S.A.

<u>Given:</u> $r = 14cm$ and $h = 20cm$



T.S.A of cylindrical drum $= 2\pi r(h+r)$

$= 2\times\frac{22}{7}\times 14(20+14)$

$= 2\times\frac{22}{\not 7}\times \overset{2}{\not{14}}\times 34$

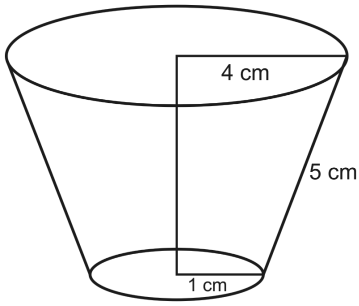
$= 44\times 68$

$= 2992$

$\therefore$ T.S.A of cylindrical drum is $2992cm^{2}$.

2. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

<u>Given:</u> Bigger radius, $\text{R} = 4\text{ cm}$, Smaller radius $\text{r} = 1\text{ cm}$, Slant height $1 = 5\text{ cm}$.



C.S.A of the Frustum $= \pi(R+r)l$

$= \frac{22}{7}(4+1)\times 5$

$= \frac{22}{7}\times 5\times 5$

$= \frac{22\times 25}{7}$

$= \frac{550}{7}$

$= 78.57cm^{2}$

3. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

<u>Given:</u> Volume of cone $= 11088\text{ cm}^3$, $\text{h} = 24\text{ cm}$, $\text{r} = ?$.

Volume of cone $= 11088$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{\not 3}\times\frac{22}{7}\times r^2\times \overset{3}{\not{24}} = 11088$

$$r^2 = \frac{\overset{63}{\cancel{11088}} \times 7}{\cancel{22} \times \cancel{8}}$$

$= 441$

$r = \sqrt{441}$

$r = 21$

$\therefore$ Radius of the cone $= 21\ cm$

4. If the ratio of radii of two spheres is $4:7$, find the ratio of their volumes.

<u>Given</u>: Ratio of two spheres $r_1 : r_2 = 4:7$

Ratio of their volume $= V_1 : V_2$

$= \frac{4}{\cancel{3}}\pi r_1^3 : \frac{4}{\cancel{3}}\pi r_2^3$

$= r_1^3 : r_2^3$

$= 4^3 : 7^3$

$= 64 : 343$

$\therefore$ The ratio of their volumes are $64 : 343$.

5. The curved surface area of a right circular cylinder of height $14\ \text{cm}$ is $88\ \text{cm}^2$. Find the diameter of the cylinder.

6. Find the diameter of a sphere whose surface area is $154\ \text{m}^2$.

7. A garden roller whose length is $3\ \text{m}$ long and whose diameter is $2.8\ \text{m}$ is rolled to level a garden. How much area will it cover in 8 revolutions?

8. Find the volume of a cylinder whose height is $2\ \text{m}$ and whose base area is $250\ \text{m}^2$.

9. If the base area of a hemispherical solid is $1386\ \text{m}^2$, then find its total surface area?

10. If the circumference of a conical wooden piece is $484\ \text{cm}$ then find its volume when its height is $105\ \text{cm}$.

11. The radius of a spherical balloon increases from $12\ \text{cm}$ to $16\ \text{cm}$ as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

12. Find the volume of the iron used to make a hollow cylinder of height $9\ \text{cm}$ and whose internal and external radii are $21\ \text{cm}$ and $28\ \text{cm}$ respectively.

13. The volumes of two cones of same base radius are $3600\ \text{cm}^3$ and $5040\ \text{cm}^3$. Find the ratio of heights.

14. The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is $7\ \text{m}$, find its height.

15. 4 persons live in a conical tent whose slant height is $19\ \text{cm}$. If each person require $22\ \text{cm}^2$ of the floor area, then find the height of the tent.

8. STATISTICS AND PROBABILITY

1. Find the range and the coefficient of range of $43.5,\ 13.6,\ 18.9,\ 38.4,\ 61.4,\ 29.8$.

Given: Largest value $L = 61.4$

Smallest value $S = 13.6$

Range $R = L - S$

$= 61.4 - 13.6$

$= 47.8$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$= \frac{61.4-13.6}{61.4+13.6}$$

$$= \frac{47.8}{75}$$

$$= 0.64$$

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Given: $S.D\ \ \sigma = 1.2$

$C.V = 25.6$

$Mean\ \overline{x} = ?$

$$C.V = \frac{\sigma}{\overline{x}} \times 100$$

$$\Rightarrow \overline{x} = \frac{\sigma}{C.V} \times 100$$

$$= \frac{1.2}{25.6} \times 100$$

$$= \frac{120}{25.6}$$

$\therefore$Mean $= 4.69$

3. A coin is tossed thrice. What is the probability of getting two consecutive tails.

$$Sample\ Space\ \ S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

$$n(S) = 8$$

$$A \rightarrow \text{getting two consecutive tails}$$

$$A = \{TTH, HTT\}$$

$$n(A) = 2$$

$\therefore$ Probability of getting two consecutive tails.

$$P(A) = \frac{n(A)}{n(S)} = \frac{\overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}}} = \frac{1}{4}$$

4. Two unbiased dice are rolled once. Find the probability of getting a doublet.

$$\text{Sample Space}\ \ S = \{(1,1),(1,2),....................(6,6)\}$$

$$n(S) = 36$$

$$A \rightarrow \text{getting a doublet}$$

$$A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$n(A) = 6$$

$\therefore$ Probability of getting a doublet is

$$P(A) = \frac{n(A)}{n(S)} = \frac{\overset{1}{\cancel{6}}}{\underset{6}{\cancel{36}}} = \frac{1}{6}$$

5. Three fair coins are tossed together. Find the probability of getting atmost two tails.

$$Sample\ space\ \ S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

$$n(S) = 8$$

$$A \rightarrow \text{getting atmost two tails}$$

$$A = \{HHH, HHT, HTH, THH, TTH, THT, HTT\}$$

$$n(A) = 7$$

$\therefore$ Probability of getting atmost two tails.

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

6. If A and B are two events such that $\mathrm{P(A)} = 0.42$, $\mathrm{P(B)} = 0.48$ and $P(\mathrm{A} \cap \mathrm{B}) = 0.16$, then find $\mathrm{P(A\, or\, B)}$.

<u>Given</u>: $\mathrm{P(A)} = 0.42$

$$\mathrm{P(B)} = 0.48$$
$$P(\mathrm{A} \cap \mathrm{B}) = 0.16$$
$$P(A \text{ or } B) = ?$$

$$\mathrm{P(A\, or\, B)} = \mathrm{P(A \cup B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \cap B)}$$
$$= 0.42 + 0.48 - 0.16$$
$$= 0.90 - 0.16$$
$$= 0.74$$

7. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $\mathrm{P}(\overline{\mathrm{A}}) + P(\overline{\mathrm{B}})$.

Given : $\mathrm{P(A \cup B)} = 0.6$

$$\mathrm{P(A \cap B)} = 0.2$$
$$\mathrm{P}(\overline{\mathrm{A}}) + P(\overline{\mathrm{B}}) = ?$$

We know that

$$\mathrm{P(A \cup B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \cap B)}$$
$$0.6 = \mathrm{P(A)} + \mathrm{P(B)} - 0.2$$
$$\mathrm{P(A)} + \mathrm{P(B)} = 0.6 + 0.2$$
$$= 0.8$$

$$\mathrm{P}(\overline{\mathrm{A}}) + P(\overline{\mathrm{B}}) = (1 - \mathrm{P(A)}) + (1 - \mathrm{P(B)})$$
$$= 1 - \mathrm{P(A)} + 1 - \mathrm{P(B)}$$
$$= 2 - \mathrm{P(A)} - \mathrm{P(B)}$$
$$= 2 - [\mathrm{P(A)} + \mathrm{P(B)}]$$
$$= 2 - 0.8$$
$$= 1.2$$

8. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events then find the probability that neither A nor B happen.

Given: $P(A) = 0.5$

$P(B) = 0.3$

$P(\text{neither } A \text{ nor } B) = ?$

$$
\begin{aligned}
P(\text{neither } A \text{ nor } B) &= P(\overline{A} \cap \overline{B}) \\
&= P(A \cup B) \\
&= 1 - P(A \cup B) \\
&= 1 - [P(A) + P(B)] \quad [\text{since A and B are mutually} \\
&= 1 - [0.5 + 0.3] \quad \text{exclusive events, } P(A \cap B) = 0] \\
&= 1 - 0.8 \\
&= 0.2
\end{aligned}
$$

9. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

10. Find the range of the distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
No. of students	0	4	6	8	2	2

11. Find the standard deviation of first 21 natural numbers.

12. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

13. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

14. What is the probability that a leap year selected at random will contain 53 saturdays.

15. From a pack of 52 cards, one card is drawn at random. Find the probability of getting a red king card.

16. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

17. Write the sample space for tossing three coins using tree diagram.

18. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is neither white nor black.

19. Two unbiased dice are rolled once. Find the probability of getting the product as a prime number.

20. Two coins are tossed together. What is the probability of getting different faces on the coins?

21. If the standard deviation of a data is 4.5 and if each value of the data is increased by 5, then find the new standard deviation.

ONE MARK QUESTIONS

1. RELATIONS AND FUNCTIONS

Multiple choice questions

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$, then $n(B)$ is

   (1) 1 (2) 2 (3) 3 (4) 6

2. $A = \{a, b, p\}, B = \{2, 3\}, C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is

   (1) 8 (2) 20 (3) 12 (4) 16

3. If $A = \{1,2\}, B = \{1,2, 3, 4\}, C = \{5,6\}$ and $D = \{5, 6, 7, 8\}$, then state which of the following statement is true.

   (1) $(A \times C) \subset (B \times D)$ (2) $(B \times D) \subset (A \times C)$

   (3) $(A \times B) \subset (A \times D)$ (4) $(D \times A) \subset (B \times A)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is

   (1) 3 (2) 2 (3) 4 (4) 8

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than $13\}$ is

   (1) $\{2,3,5,7\}$ (2) $\{2,3,5,7,11\}$ (3) $\{4,9,25,49,121\}$ (4) $\{1,4,9,25,49,121\}$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal, then (a, b) is

   (1) $(2, -2)$ (2) $(5,1)$ (3) $(2,3)$ (4) $(3, -2)$

7. Let $n(A) = m$ and $n(B) = n$, then the total number of non-empty relations that can be defined from A to B is

   (1) m^n (2) n^m (3) $2^{mn} - 1$ (4) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively

   (1) $(8,6)$ (2) $(8,8)$ (3) $(6,8)$ (4) $(6,6)$

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f : A \to B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a

   (1) Many-one function (2) Identity function

   (3) One-to-one function (4) Into function

10. If $f(x) = 2x^2$ and $g(x)=\frac{1}{3x}$, then $f \circ g$ is

(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$ (3) $\frac{2}{9x^2}$ (4) $\frac{1}{6x^2}$

11. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to

(1) 7 (2) 49 (3) 1 (4) 14

12. Let f and g be two functions given by $f = \{(0,1),(2, 0),(3,-4),(4,2),(5,7)\}$

$g = \{(0,2),(1, 0),(2, 4),(-4,2),(7, 0)\}$, then the range of $f \circ g$ is

(1) $\{0,2,3,4,5\}$ (2) $\{-4,1,0,2,7\}$ (3) $\{1,2,3,4,5\}$ (4) $\{0,1,2\}$

13. Let $f(x) = \sqrt{1+x^2}$, then

(1) $f(xy) = f(x).f(y)$ (2) $f(xy) \geq f(x).f(y)$
(3) $f(xy) \leq f(x).f(y)$ (4) None of these

14. If $g = \{(1,1),(2, 3),(3,5),(4,7)\}$ is a function given by $g(x) = \alpha x + \beta$, then the values of α and β are

(1) $(-1,2)$ (2) $(2,-1)$ (3) $(-1,-2)$ (4) $(1,2)$

15. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is

(1) linear (2) cubic (3) reciprocal (4) quadratic

16. If $f : R \to R$ defined by $f(x) = x^2 + 2$, then the pre-images of 27 are

(1) $5,-5$ (2) $\sqrt{5},-\sqrt{5}$ (3) $5,0$ (4) 0.5

17. If $f\left(x-\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, then $f(x) =$

(1) x^2+2 (2) x^2-2 (3) $x^2+\frac{1}{x^2}$ (4) $x^2-\frac{1}{x^2}$

18. If $A=\{a,b,c\}$, $B=\{2,3\}$ and $C=\{a,b,c,d\}$, then $n[(A\cap C)\times B]$ is

(1) 4 (2) 8 (3) 6 (4) 12

19. If the ordered pairs $(a,-1)$ and $(5,b)$ belong to $\left\{\begin{pmatrix} x, y \\ y \end{pmatrix} = 2x+3\right\}$, then the values of a and b are

(1) $-13,2$ (2) $2,13$ (3) $2,-13$ (4) $-2,13$

20. The function $f:\mathbb{N}\to\mathbb{N}$ is defined by $f(x)=2x.$ Then the function f is

(1) Not one-one but onto (2) one-one but not onto
(3) One-one and onto (4) not one-one and not onto

21. If $f(x)=x+1,$ then $f(f(f(y+2)))$ is

(1) $y+3$ (2) $y+5$ (3) $y+7$ (4) $y+9$

22. If $f(x)=mx+n,$ where m and n are integers, $f(-2)=7$ and $f(3)=2,$ then m and n are equal to

(1) $-1,5$ (2) $-1,-5$ (3) $1,-9$ (4) $1,9$

23. The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C)=\frac{9C}{5}+32.$ The Fahrenheit degree is $95,$ then the value of C will be

(1) 37 (2) 36 (3) 35 (4) 29

24. If $f(x)=ax-2, g(x)=2x-1$ and $f\circ g=g\circ f$, then the value of a is

(1) -3 (2) 3 (3) $\frac{1}{3}$ (4) 13

25. If $f(x)=\frac{1}{x}$ and $g(x)=\frac{1}{x^3},$ then $f\circ g\circ f(y)$ is

(1) $\frac{1}{y^8}$ (2) $\frac{1}{y^6}$ (3) $\frac{1}{y^4}$ (4) $\frac{1}{y^3}$

26. If $f(x)=2-3x,$ then $f\circ f(1-x)$ is

(1) $9x-5$ (2) $5x-9$ (3) $5x+9$ (4) $5-9x$

27. If $f(x)+f(1-x)=2$ then $f\left(\frac{1}{2}\right)$ is

(1) 1 (2) -1 (3) 5 (4) -9

28. If f is a constant function of value $\frac{1}{10},$ then the value of $f(1)+f(2)+......+f(100)$ is

(1) $\frac{1}{10}$ (2) 10 (3) 100 (4) $\frac{1}{100}$

29. If $f(x)=\frac{x+1}{x-2}$ and $g(x)=\frac{1+2x}{x-1},$ then $f\circ g(x)$ is

(1) constant function (2) Identity function (3) quadratic function (4) cubic function

30. If f is a identity function, then the value of $f(1)-2f(2)+f(3)$ is

(1) 1 (2) 0 (3) -1 (4) -3

2. NUMBERS AND SEQUENCES

1. Euclid's division lemma states that for positive integers △ and b, there exist unique integers △ and r such that $a = bq + r$, where r must satisfy

   (1) $1 < r < b$ (2) $0 < r < b$ (3) $0 \leq r < b$ (4) $0 < r \leq b$

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are

   (1) 0, 1, 8 (2) 1, 4, 8 (3) 0, 1, 3 (4) 1, 3, 5

3. If the H.C.F of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is

   (1) 4 (2) 2 (3) 1 (4) 3

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is

   (1) 1 (2) 2 (3) 3 (4) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

   (1) 2025 (2) 5220 (3) 5025 (4) 2520

6. $7^{4k} \equiv$ ____ (mod 100)

   (1) 1 (2) 2 (3) 3 (4) 4

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is

   (1) 3 (2) 5 (3) 8 (4) 11

8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P

   (1) 4551 (2) 10091 (3) 7881 (4) 13531

9. If 6 times of 6^{th} term of an A.P is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is

   (1) 0 (2) 6 (3) 7 (4) 13

10. An A.P consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is

    (1) 16m (2) 62m (3) 31m (4) $\frac{31}{2}$m

11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?

    (1) 6 (2) 7 (3) 8 (4) 9

12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \quad + 2^0$ which of the following is true?

    (1) B is 2^{64} more than A (2) A and B are equal
    (3) B is larger than A by 1 (4) A is larger than B by 1

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, ...$ is

(1) $\frac{1}{24}$ (2) $\frac{1}{27}$ (3) $\frac{2}{3}$ (4) $\frac{1}{81}$

14. If the sequence $t_1, t_2, t_3, ...$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, ...$ is

(1) a Geometric Progression
(2) an Arithmetic Progression
(3) neither an Arithmetic Progression nor a Geometric Progression
(4) a constant sequence

15. The value of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$

(1) 14400 (2) 14200 (3) 14280 (4) 14520

16. What is the HCF of the least prime number and the least composite number?

(1) 1 (2) 2 (3) 3 (4) 4

17. If $'a'$ and $'b'$ are two positive integers where $a > b$ and $'b'$ is a factor of $'a'$ then HCF of (a, b) is

(1) b (2) a (3) ab (4) $\frac{a}{b}$

18. If m and n are co-prime numbers, then m^3 and n^3 are

(1) co-prime (2) Not co-prime (3) even (4) odd

19. If 3 is the least prime factor of number a and 7 is the least prime factor of b then the least prime factor of $a + b$ is

(1) $a + b$ (2) 2 (3) 5 (4) 10

20. The remainder when the difference between 60002 and 601 is divided by 6 is

(1) 2 (2) (3) 0 (4) 3

21. $44 \equiv 8 (\text{mod } 12)$, $113 \equiv 5 (\text{mod } 12)$, thus $44 \times 113 \equiv$ ____$(\text{mod } 12)$

(1) 4 (2) 3 (3) 2 (4) 1

22. Given $a_1 = -1$ and $a_n = \frac{a_{n-1}}{n+2}$ then a_4 is

(1) $-\frac{1}{20}$ (2) $-\frac{1}{4}$ (3) $-\frac{1}{840}$ (4) $-\frac{1}{120}$

23. The first term of an A.P. whose 8^{th} and 12^{th} terms are 39 and 59 respectively is

(1) 5 (2) 6 (3) 4 (4) 3

24. In the arithmetic series, $S_n = k + 2k + 3k + ... + 100,$ k is a positive integer and k is a factor of 100, then S_n is

(1) $5000 + \frac{5}{k}$ (2) $\frac{5000}{k} + 50$ (3) $\frac{1000}{k} + 10$ (4) $1000 + \frac{10}{k}$

25. How many terms are there in the G.P. $5, 20, 80, 320, ..., 20480$?

(1) 5 (2) 6 (3) 7 (4) 9

26. If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then $a(q-r) + b(r-p) + c(p-q)$ is

(1) 0 (2) $a+b+c$ (3) $p+q+r$ (4) pqr

27. Sum of infinite terms of a G.P. is 12 and the first term is 8. What is the fourth term of the G.P.?

(1) $\frac{8}{27}$ (2) $\frac{4}{27}$ (3) $\frac{8}{20}$ (4) $\frac{1}{3}$

28. A square is drawn by joining the mid points of the sides of a given square in the same way and this process continues indefinitely. If the side of the first square is $4\,\text{cm}$, then the sum of the areas of all the square is

(1) $8\,\text{cm}^2$ (2) $16\,\text{cm}^2$ (3) $32\,\text{cm}^2$ (4) $64\,\text{cm}^2$

29. A boy saves ₹1 on the first day ₹2 on the second day, ₹4 on the third day and so on. How much did the boy will save up to 20 days?

(1) $2^{19} + 1$ (2) $2^{19} - 1$ (3) $2^{20} - 1$ (4) $2^{21} - 1$

30. The sum of first 'n' terms of the series $a, 3a, 5a, ...$ is

(1) na (2) $(2n-1)a$ (3) $n^2 a$ (4) $n^2 a^2$

31. If p, q, r, x, y, z are in the A.P. then $5p+3, 5q+3, 5r+3, 5x+3, 5y+3, 5z+3$ form

(1) a G.P (2) A.P.

(3) Constant sequence (4) neither an A.P. nor a G.P.

32. In an A.P. if the p^{th} term is 'q' and the q^{th} term is 'p' then its n^{th} term is

(1) $p+q-n$ (2) $p+q+n$ (3) $p-q+n$ (4) $p-q-n$

33. Sum of first 'n' terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + ...$ is

(1) $\frac{n(n+1)}{2}$ (2) $\sqrt{n}$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1

3. ALGEBRA

1. A system of three linear equations in three variables is inconsistent if their planes

   (1) intersect only at a point (2) intersect in a line
   (3) coincides with each other (4) do not intersect

2. The solution of the system $x + y - 3x = -6$, $-7y + 7z = 7$, $3z = 9$ is

   (1) $x = 1, y = 2, z = 3$ (2) $x = -1, y = 2, z = 3$
   (3) $x = -1, y = -2, z = 3$ (4) $x = 1, y = 2, z = 3$

3. If $(x-6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$, then the value of k is

   (1) 3 (2) 5 (3) 6 (4) 8

4. $\dfrac{3y-3}{y} \div \dfrac{7y-7}{3y^2}$ is

   (1) $\dfrac{9y}{7}$ (2) $\dfrac{9y^3}{(21y-21)}$ (3) $\dfrac{-21y^2-42y+21}{3y^3}$ (4) $\dfrac{7(y^2-2y+1)}{y^2}$

5. $y^2 + \dfrac{1}{y^2}$ is not equal to

   (1) $\dfrac{y^4+1}{y^2}$ (2) $\left(y+\dfrac{1}{y}\right)^2$ (3) $\left(y-\dfrac{1}{y}\right)^2 + 2$ (4) $\left(y+\dfrac{1}{y}\right)^2 - 2$

6. $\dfrac{x}{x^2-25} - \dfrac{8}{x^2+6x+5}$ gives

   (1) $\dfrac{x^2-7x+40}{(x-5)(x+5)}$ (2) $\dfrac{x^2+7x+40}{(x-5)(x+5)(x+1)}$

   (3) $\dfrac{x^2-7x+40}{(x^2-25)(x+1)}$ (4) $\dfrac{x^2+10}{(x^2-25)(x+1)}$

7. The square root of $\dfrac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to

   (1) $\left(\dfrac{16x^2z^4}{5y^2}\right)$ (2) $\left(\dfrac{16y^2}{x^2z^4}\right)$ (3) $\left(\dfrac{16y}{5xz^2}\right)$ (4) $\left(\dfrac{16xz^2}{5y}\right)$

8. Which of the following should be added to make $x^4 + 64$ a perfect square

   (1) $4x^2$ (2) $16x^2$ (3) $8x^2$ (4) $-8x^2$

9. The solution of $(2x-1)^2 = 9$ is equal to

   (1) -1 (2) 2 (3) $-1, 2$ (4) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are

(1) $100, 120$ (2) $10, 12$ (3) $-120, 100$ (4) $12, 10$

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in ______.

(1) $A.P$ (2) $G.P$ (3) Both $A.P$ and $G.P$ (4) None of these

12. Graph of a linear polynomial is a

(1) straight line (2) circle (3) parabola (4) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is.

(1) 0 (2) 1 (3) 0 or 1 (4) 2

14. For the given matrix $A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ then the order of matrix A^T is.

(1) 2×3 (2) 3×2 (3) 3×4 (4) 4×3

15. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have

(1) 3 (2) 4 (3) 2 (4) 5

16. If number of columns and rows are not equal in a matrix then it is said to be a

(1) diagonal matrix (2) rectangular matrix
(3) square matrix (4) identity matrix

17. Transpose of a column matrix is

(1) unit matrix (2) diagonal matrix
(3) column matrix (4) row matrix

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$

(1) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (4) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. Which of the following can be made from the given matrices

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, (i) A^2 (ii) B^2 (iii) AB (iv) BA

(1) (i) and (ii) only (2) (ii) and (iii) only
(3) (ii) and (iv) only (4) all of these

20. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?

(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$

(iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $ABC = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

(1) (i) and (ii) only (2) (ii) and (iii) only
(3) (iii) and (iv) only (4) all of these

21. Which of the following are linear equation in three variables

(i) $2x = z$ (ii) $2\sin x + y\cos y + z\tan z = 2$ (iii) $x = 2y^2 + z = 3$ (iv) $x - y - z = 7$ is

(1) (i) and (iii) only (2) (i) and (iv) only (3) (iv) only (4) All

22. Graphically a infinite number of solutions represents

(1) three planes with no point in connection
(2) three planes intersecting at a single point
(3) three planes intersecting in a line or coinciding with one another
(4) None

23. Which of the following is correct

(i) Every polynomial has finite number of multiples
(ii) LCM of two polynomials of degree 2 may be a constant
(iii) HCF of 2 polynomials may be a constant
(iv) Degree of HCF of two polynomials is always less then degree of LCM.

(1) (i) and (ii) (2) (iii) and (iv) (3) (iii) only (4) (iv) only

24. The HCF of two polynomials $p(x)$ and $q(x)$ is $2x(x+2)$ and LCM is $24x(x+2)^2(x-2)$. If $p(x) = 8x^3 + 32x^2 + 32x$ then $q(x)$ is equal to

(1) $4x^3 - 16x$ (2) $6x^3 - 24x$ (3) $12x^3 + 24x$ (4) $12x^3 - 24x$

25. Consider the following statements:

(i) The HCF of $x + y$ and $x^8 - y^8$ is $x + y$ (ii) The HCF of $x + y$ and $x^8 + y^8$ is $x + y$
(iii) The HCF of $x - y$ and $x^8 + y^8$ is $x - y$ (iv) The HCF of $x - y$ and $x^8 - y^8$ is $x - y$

Which of the statements given above are correct?

(1) (i) and (ii) (2) (ii) and (iii) (3) (i) and (iv) (4) (ii) and (iv)

26. For what set of values $\frac{x^2+5x+6}{x^2+8x+15}$ is undefined

(1) $-3,-5$ (2) -5 (3) $-2,-5$ (4) $-2,-3$

27. $\frac{x^2+7x+12}{x^2+8x+15} \times \frac{x^2+5x}{x^2+6x+8}$

(1) $x+2$ (2) $\frac{x}{x+2}$ (3) $\frac{35x^2+6}{48x^2+120}$ (4) $\frac{1}{x+2}$

28. If $\frac{p}{q}=a$ then $\frac{p^2+q^2}{p^2-q^2}$

(1) $\frac{a^2+1}{a^2-1}$ (2) $\frac{1+a^2}{1-a^2}$ (3) $\frac{1-a^2}{1+a^2}$ (4) $\frac{a^2-1}{a^2+1}$

29. The square root of $4m^2-24m+36=0$ is

(1) $4(m-3)$ (2) $2(m-3)$ (3) $(2m-3)^2$ (4) $(m-3)$

30. The real roots of the quadratic equation $x^2-x-1=0$ are

(1) $1,1$ (2) $-1,1$ (3) $\frac{1+\sqrt{5}}{2},\frac{1-\sqrt{5}}{2}$ (4) No real roots

31. The product of the sum and product of roots of equation $(a^2-b^2)x^2-(a+b)x+(a^3-b^3)=0$ is

(1) $\frac{a^2+ab+b^2}{a-b}$ (2) $\frac{a+b}{a-b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{a-b}{a^2+ab+b^2}$

32. A quadratic polynomial whose one zero is 5 and sum of the zeroes 0 is given by

(1) x^2-25 (2) x^2-5 (3) x^2-5x (4) x^2-5x+5

33. Axis of symmetry in terms of vertical line separates parabola into

(1) 3 equal halves (2) 5 equal halves (3) 2 equal halves (4) 4 equal halves

34. The parabola $y=-3x^2$ is

(1) Open upward (2) Open downward (3) Open rightward (4) Open leftward

35. Choose the correct answer

(i) Every scalar matrix is an identity matrix (ii) Every identity matrix is a scalar matrix
(iii) Every diagonal matrix is an identity matrix (iv) Every null matrix is a scalar matrix

(1) (i) and (iii) only (2) (ii) only (3) (iv) only (4) (ii) and (iv) only

36. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}$ then, $B=$

(1) $\begin{bmatrix}8 & -1 & -2\\ -1 & 10 & -1\end{bmatrix}$ (2) $\begin{bmatrix}8 & -1 & 2\\ -1 & 10 & -1\end{bmatrix}$ (3) $\begin{bmatrix}8 & 1 & 2\\ 1 & 10 & 1\end{bmatrix}$ (4) $\begin{bmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{bmatrix}$

37. If $(4 \quad 3 \quad 2)\begin{bmatrix}1\\ -2\\ x\end{bmatrix}=(6)$ then x is

(1) 4 (2) 3 (3) 2 (4) 1

38. If $A=\begin{pmatrix}y & 0\\ 3 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ then $A^2=16I$ for

(1) $y=4$ (2) $y=5$ (3) $y=-4$ (4) $y=16$

39. If P and Q are matrices, then which of the following is true?

(1) $PQ \neq QP$ (2) $(P^T)^T \neq P$ (3) $P+Q \neq Q+P$ (4) All are true

40. If $A=\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}_{3\times 2}$, $B=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}_{2\times 3}$ then which of the following products can be made from these matrices (i) A^2 (ii) B^2 (iii) AB (iv) BA

(1) (i) only (2) (ii) and (iii) only (3) (iii) and (iv) only (4) All the above

4. GEOMETRY

1. If in triangles ABC and EDF , $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

   (1) $\angle B = \angle E$ (2) $\angle A = \angle D$ (3) $\angle B = \angle D$ (4) $\angle A = \angle F$

2. In $\Delta LMN, \angle L = 60^\circ$, $\angle M = 50^\circ$. If $\Delta LMN \sim \Delta PQR$, then value of $\angle R$ is

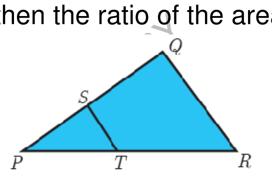
   (1) 40° (2) 70° (3) 30° (4) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90^\circ$. and $AC = 5cm$, then AB is

   (1) 2.5cm (2) 5cm (3) 10cm (4) $5\sqrt{2}\text{ cm}$

4. In the given figure $ST \parallel QR$, $PS = 2\text{cm}$ and $SQ = 3\text{cm}$, then the ratio of the area of ΔPQR to the area of ΔPST is

   (1) $25:4$ (2) $25:7$
   (3) $25:11$ (4) $25:13$



5. The perimeters of two similar triangles ΔABC and ΔPQR are 36cm and 24cm respectively. If $PQ = 10\text{cm}$, then the length of AB is

   (1) $6\frac{2}{3}\text{cm}$ (2) $\frac{10\sqrt{6}}{3}\text{cm}$ (3) $66\frac{2}{3}\text{cm}$ (4) 15cm

6. If in ΔABC, $DE \parallel BC$. $AB = 3.6\text{cm}$, $AC = 2.4\text{cm}$ and $AD = 2.1\text{cm}$, then the length of AE is

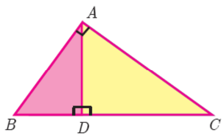
   (1) 1.4cm (2) 1.8cm (3) 1.2cm (4) 1.05cm

7. In a ΔABC, AD is the bisector of $\angle BAC$. If $AB = 8\text{cm}$, $BD = 6\text{cm}$ $DC = 3\text{cm}$. The length of the side AC is

   (1) 6cm (2) 4cm (3) 3cm (4) 8cm

8. In the adjacent figure $\angle BAC = 90^\circ$ and $AD \perp BC$ then

   (1) $BD.CD = BC^2$ (2) $AB.AC = BC^2$
   (3) $BD.CD = AD^2$ (4) $AB.AC = AD^2$

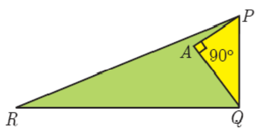


9. Two poles of heights 6m and 11m stand vertically on a plane ground. If the distance between their feet is 12m, what is the distance between their tops?

   (1) 13m (2) 14m (3) 15m (4) 12.8m

10. In the given figure, $PR = 26\text{cm}$, $QR = 24\text{cm}$, $\angle PAQ = 90^\circ$, $PA = 6\text{cm}$ and $QA = 8\text{cm}$. Find $\angle PQR$

    (1) 80° (2) 85°
    (3) 75° (4) 90°



11. A tangent is perpendicular to the radius at the

(1) centre (2) point of contact (3) infinity (4) chord

12. How many tangents can be drawn to the circle from an exterior point?

(1) one (2) two (3) infinite (4) zero

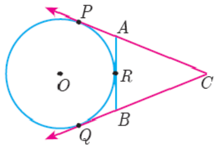
13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^\circ$, then the value of $\angle AOB$ is

(1) 100° (2) 110° (3) 120° (4) 130°

14. In figure, CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11\text{cm}$ and $BC = 7\,\text{cm}$, then the length of BR is

(1) 6cm (2) 5cm

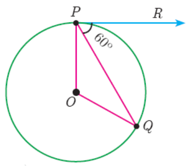
(3) 8cm (4) 4cm



15. In figure, if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is

(1) 120° (2) 100°

(3) 110° (4) 90°



16. If triangle PQR is similar to triangle LMN, such that $4PQ = LM$ and $QR = 6\,cm$, then MN is equal to

(1) 12 cm (2) $24\,\text{cm}$ (3) $10\,\text{cm}$ (4) $36\,\text{cm}$

17. In ΔABC, $DE \parallel AC$, $DE = a$, $AC = b$, $BE = x$ and $CE = y$, then which of the following is true.

(1) $x = \dfrac{ay}{b+a}$ (2) $x = \dfrac{a+b}{ay}$ (3) $x = \dfrac{ay}{b-a}$ (4) $\dfrac{x}{y} = \dfrac{a}{b}$

18. If S and T are points on sides PQ and PR respectively of ΔPQR. If $PS = 3$ cm, $SQ = 6$ cm $PT = 5$ cm, $TR = 10$ cm, then QR is equal to

(1) $4ST$ (2) $5ST$ (3) $3ST$ (4) $3QR$

19. In ΔABC, $DE \parallel BC$, if $BD = x-3$, $BA = 2x$, $CE = x-2$ and $AC = 2x+3$, then the value of x is

(1) 3 (2) 6 (3) 9 (4) 12

20. The ratio of the areas of two similar triangles is equal to

(1) The ratio of their corresponding sides
(2) The cube of the ratio of their corresponding sides
(3) The ratio of their corresponding altitudes
(4) The square of the ratio of their corresponding sides

21. In ΔABC, AD bisects $\angle A$, $AB = 4$ cm, $BD = 6$ cm, $DC = 8$ cm, then the value of AC is

(1) $\frac{16}{3}$ cm (2) $\frac{32}{3}$ cm (3) $\frac{3}{16}$ cm (4) $\frac{1}{2}$ cm

22. In a triangle, the internal bisector of an angle bisects the opposite side. The nature of the triangle is.

(1) right angle (2) equilateral (3) scalene (4) isosceles

23. The height of an equilateral triangle of side a is

(1) $\frac{a}{2}$ (2) $\sqrt{3a}$ (3) $\frac{\sqrt{3}}{2}a$ (4)

24. The perimeter of a right triangle is 36 cm. Its hypotenuse is 15 cm, then the area of the triangle is

(1) 108 cm^2 (2) 54 cm^2 (3) 27 cm^2 (4) 216 cm^2

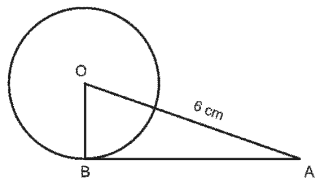
25. A line which intersects a circle at two distinct points is called

(1) Point of contact (2) secant (3) diameter (4) tangent

26. If the angle between two radii of a circle is 130°, then angle between the tangents at the end of the radii is

(1) 50° (2) 90° (3) 40° (4) 70°

27. In figure, $\angle OAB = 60^\circ$ and $OA = 6$ cm, then radius of the circle is



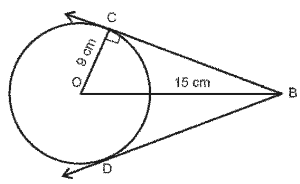
(1) $\frac{3}{2}\sqrt{3}$ cm (2) 2 cm

(3) $3\sqrt{3}$ cm (4) $2\sqrt{3}$

28. In the given figure, if $OC = 9$ cm and $OB = 15$ cm, then $OB + BD$ is equal to

(1) 23 cm (2) 24 cm

(3) 27 cm (4) 30 cm



29. Two concentric circles of radii a and b, where $a > b$ are given. The length of the chord of the larger circle which touches the smaller circle is

(1) $\sqrt{a^2 - b^2}$ (2) $2\sqrt{a^2 - b^2}$ (3) $\sqrt{a^2 + b^2}$ (4) $2\sqrt{a^2 + b^2}$

30. Three circles are drawn with the vertices of a triangle as centres such that each circle touches the other two if the sides of the triangle are 2 cm, 3 cm and 4 cm, then the diameter of the smallest circle is

(1) 1 cm (2) 3 cm (3) 5 cm (4) 4 cm

5. COORDINATE GEOMETRY

1. The area of triangle formed by the points $(-5,0)$, $(0,-5)$ and $(5,0)$ is

   (1) 0 sq.units (2) 25 sq.units (3) 5 sq.units (4) none of these

2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is

   (1) $x=10$ (2) $y=0$ (3) $x=0$ (4) $y=10$

3. The straight line given by the equation $x=11$ is

   (1) parallel to X axis (2) parallel to Y axis
   (3) passing through the origin (4) passing through the point $(0,11)$

4. If $(5,7),(3,p)$ and $(6,6)$ are collinear, then the value of p is

   (1) 3 (2) 6 (3) 9 (4) 12

5. The point of intersection of $3x-y=4$ and $x+y=8$ is

   (1) $(5,3)$ (2) $(2,4)$ (3) $(3,5)$ (4) $(4,4)$

6. The slope of the line joining $(12,3)$, $(4,a)$ is $\frac{1}{8}$. Then value of $'a'$ is

   (1) 1 (2) 4 (3) -5 (4) 2

7. The slope of the line which is perpendicular to the line joining the points $(0,0)$ and $(-8,8)$ is

   (1) -1 (2) 1 (3) $\frac{1}{3}$ (4) -8

8. If the slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is

   (1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 0

9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissa is 5 then the equation of the line AB is

   (1) $8x+5y=40$ (2) $8x-5y=40$ (3) $x=8$ (4) $y=5$

10. The equation of a line passing through the origin and perpendicular to the line $7x-3y+4=0$ is

    (1) $7x-3y+4=0$ (2) $3x-7y+4=0$ (3) $3x+7y=0$ (4) $7x-3y=0$

11. Consider four straight lines

    (i) $l_1: 3y=4x+5$ (ii) $l_2: 4y=3x-1$ (iii) $l_3: 4y+3x=7$ (iv) $l_4: 4x+3y=2$

    Which of the following statement is true?

    (1) l_1 and l_2 are perpendicular (2) l_1 and l_4 are parallel
    (3) l_2 and l_4 are perpendicular (4) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21.$ Which of the following is true?

(1) The slope is 0.5 and the y intercept is 2.6
(2) The slope is 5 and the y intercept is 1.6
(3) The slope is 0.5 and the y intercept is 1.6
(4) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium, it is necessary to show

(1) Two sides are parallel. (2) Two parallel and two non-parallel sides.
(3) Opposite sides are parallel. (4) All sides are of equal length.

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

(1) The slopes of two sides (2) The slopes of two pair of opposite sides
(3) The lengths of all sides (4) Both the lengths and slopes of two sides.

15. $(2,1)$ is the point of intersection of two lines.

(1) $x - y - 3 = 0;\ 3x - y - 7 = 0$ (2) $x + y = 3;\ 3x + y = 7$
(3) $3x + y = 3;\ x + y = 7$ (4) $x + 3y - 3 = 0;\ x - y - 7 = 0$

16. The ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is internally divided by $(-1,6)$ is

(1) $7:2$ (2) $3:2$ (3) $2:7$ (4) $5:3$

17. If the points $(0,0),(a,0)$ and $(0,b)$ are collinear, then

(1) $a = b$ (2) $a + b = 0$ (3) $ab = 0$ (4) $a \neq b$

18. If the mid point of the segment joining $A\left(\frac{x}{2}, \frac{y+1}{2}\right)$ and $B(x+1,\ y-3),\ C(5,-2),$ then the value of x, y is

(1) $(6,-1)$ (2) $(-6,1)$ (3) $(-2,1)$ (4) $(3,5)$

19. The area of triangle formed by the points $(a,b+c),(b,c+a)$ and $(c,a+b)$ is

(1) $a+b+c$ (2) abc (3) $(a+b+c)^2$ (4) 0

20. The four vertices of a quadrilateral are $(1,2),\ (-5,6),\ (7,-4)$ and $(k,-2)$ taken in order. If the area of quadrilateral is zero, then the value of k is

(1) 4 (2) -2 (3) 6 (4) 3

21. The equation of the line passing through the point $(5,3)$ which is parallel to the y axis is

(1) $y = 5$ (2) $y = 3$ (3) $x = 5$ (4) $x = 3$

22. The slope of the line $2y = x + 8$ is

(1) $\frac{1}{2}$ (2) 1 (3) 8 (4) 2

23. The value of p, given that the line $\frac{y}{2} = x - p$ passes through the point $(-4, 4)$ is

(1) -4 (2) -6 (3) 0 (4) 8

24. The slope and the $y-$intercept of the line $3y - \sqrt{3}x + 1 = 0$ is

(1) $\frac{1}{\sqrt{3}}, \frac{-1}{3}$ (2) $-\frac{1}{\sqrt{3}}, \frac{-1}{3}$ (3) $\sqrt{3}, 1$ (4) $-\sqrt{3}, 3$

25. The value of $'a'$, if the lines $7y = ax + 4$ and $2y = 3 - x$ are parallel is

(1) $a = \frac{7}{2}$ (2) $a = -\frac{7}{2}$ (3) $a = \frac{2}{7}$ (4) $a = -\frac{7}{2}$

26. A line passing through the point $(2, 2)$ and the axes enclose an area α The intercepts on the axes made by the line are given by the roots of

(1) $x^2 - 2\alpha x + \alpha = 0$ (2) $x^2 + 2\alpha x + 2\alpha = 0$ (3) $x^2 - \alpha x + 2\alpha = 0$ (4) None of these

27. The equation of the line passing through the point $(0, 4)$ and is parallel to the line $3x + 5y + 15 = 0$ is

(1) $3x + 5y + 15 = 0$ (2) $3x + 5y - 20 = 0$
(3) $2x + 7y - 20 = 0$ (4) $4x + 3y - 15 = 0$

28. In triangle ABC, right angled at B, if the side BC is parallel to x axis, then the slope of AB is

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) 1 (4) Not defined

29. The $y-$intercept of the line $3x - 4y + 8 = 0$ is

(1) $-\frac{8}{3}$ (2) $\frac{3}{8}$ (3) 2 (4) $\frac{1}{2}$

30. The lines $y = 5x - 3$, $y = 2x + 9$ intersect at A. The coordinates of A are

(1) $(2, 7)$ (2) $(2, 3)$ (3) $(4, 17)$ (4) $(-4, 23)$

6. TRIGONOMETRY

1. The value of $\sin^2\theta + \dfrac{1}{1+\tan^2\theta}$ is equal to

   (1) $\tan^2\theta$ (2) 1 (3) $\cot^2\theta$ (4) 0

2. $\tan\theta\operatorname{cosec}^2\theta - \tan\theta$ is equal to

   (1) $\sec\theta$ (2) $\cot^2\theta$ (3) $\sin\theta$ (4) $\cot\theta$

3. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha,$ then the value of k is equal to

   (1) 9 (2) 7 (3) 5 (4) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b,$ then the value of $b(a^2 - 1)$ is equal to

   (1) $2a$ (2) $3a$ (3) 0 (4) $2ab$

5. If $5x = \sec\theta$ and $\dfrac{5}{x} = \tan\theta,$ then $x^2 - \dfrac{1}{x}$ is equal to

   (1) 25 (2) $\dfrac{1}{25}$ (3) 5 (4) 1

6. If $\sin\theta = \cos\theta,$ then $2\tan^2\theta + \sin^2\theta - 1$ is equal to

   (1) $\dfrac{-3}{2}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{2}{3}$ (4) $\dfrac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta,$ then

   (1) $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ (2) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ (3) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (4) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to

   (1) 0 (2) 1 (3) 2 (4) -1

9. $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q,$ then $p^2 - q^2$ is equal to

   (1) $a^2 - b^2$ (2) $b^2 - a^2$ (3) $a^2 + b^2$ (4) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1,$ then the angle of elevation of the sun has measure

    (1) 45° (2) 30° (3) 90° (4) 60°

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the tower is 60°. The height of the tower (in metres) is equal to

    (1) $\sqrt{3}b$ (2) $\dfrac{b}{3}$ (3) $\dfrac{b}{2}$ (4) $\dfrac{b}{\sqrt{3}}$

12. A tower is $60\,\text{m}$ height. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to

(1) $41.92\,\text{m}$ (2) $43.92\,\text{m}$ (3) $43\,\text{m}$ (4) $45.6\,\text{m}$

13. The angle of depression of the top and bottom of $20\,\text{m}$ tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in meters) is

(1) $20, 10\sqrt{3}$ (2) $30, 5\sqrt{3}$ (3) $20, 10$ (4) $30, 10\sqrt{3}$

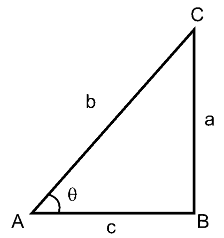
14. Two persons are standing $'x'$ metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(1) $\sqrt{2}x$ (2) $\dfrac{x}{2\sqrt{2}}$ (3) $\dfrac{x}{\sqrt{2}}$ (4) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is

(1) $\dfrac{h(1+\tan\beta)}{1-\tan\beta}$ (2) $\dfrac{h(1-\tan\beta)}{1+\tan\beta}$ (3) $h\tan(45^\circ-\beta)$ (4) None of these

16. From the figure, the value of $\operatorname{cosec}\theta+\cot\theta$ is



(1) $\dfrac{a+b}{c}$ (2) $\dfrac{c}{a+b}$

(3) $\dfrac{b+c}{a}$ (4) $\dfrac{b}{a+c}$

17. $(\sec A+\tan A)(1-\sin A)$ is equal to

(1) $\sec A$ (2) $\sin A$ (3) $\operatorname{cosec} A$ (4) $\cos A$

18. If $x=r\sin\theta\cos\varphi, y=r\sin\theta\sin\varphi$ and $z=r\cos\theta,$ then, $x^2+y^2+z^2$ is equal to

(1) r (2) r^2 (3) $\dfrac{r^2}{2}$ (4) $2r^2$

19. If $\cos\theta+\cos^2\theta=1$ then $\sin^2\theta+\sin^4\theta$ is equal to

(1) 1 (2) 0 (3) -1 (4) None of these

20. If $\tan\theta+\cot\theta=3$ then $\tan^2\theta+\cot^2\theta$ is equal to

(1) 4 (2) 7 (3) 6 (4) 9

21. If $m\cos\theta + n\sin\theta = a$ and $m\sin\theta - n\cos\theta = b$ then $a^2 + b^2$ is equal to

(1) $m^2 - n^2$ (2) $m^2 + n^2$ (3) $m^2 n^2$ (4) $n^2 - m^2$

22. $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1}$ is equal to

(1) $2\tan\theta$ (2) $2\sec\theta$ (3) $2\cos ec\theta$ (4) $2\tan\theta\sec\theta$

23. The value of $\left(\frac{3}{\cot^2\theta} - \frac{3}{\cos^2\theta}\right)$ is equal to

(1) $\frac{1}{3}$ (2) 3 (3) 0 (4) -3

24. If $\sin(\alpha + \beta) = 1$, then $\cos(\alpha - \beta)$ can be reduced to

(1) $\sin\alpha$ (2) $\cos\beta$ (3) $\sin 2\beta$ (4) $\cos 2\beta$

25. If $x = a\sec\theta$ and $y = b\tan\theta$, then $b^2x^2 - a^2y^2$ is equal to

(1) ab (2) $a^2 - b^2$ (3) $a^2 + b^2$ (4) a^2b^2

26. The angle of elevation of the top of tree from a point at a distance of $250m$ from its base is 60° with the horizontal, then the height of the wall is

(1) $150m$ (2) $250\sqrt{3}m$ (3) $\frac{250}{\sqrt{3}}m$ (4) $200\sqrt{3}m$

27. The angle of depression of a boat from a $50\sqrt{3}m$ high bridge is 30°. The horizontal distance of the boat from the bridge is

(1) $150m$ (2) $150\sqrt{3}m$ (3) $60m$ (4) $60\sqrt{3}m$

28. A Ladder of length $14m$ just reaches the top of a wall. If the ladder makes an angle of 60° with the horizontal, then the height of the wall is

(1) $14\sqrt{3}$ m (2) $28\sqrt{3}$ m (3) $7\sqrt{3}$ m (4) $35\sqrt{3}$ m

29. The top of two poles of height 18.5 m and 7 m are connected by a wire. If the wire makes an angle of measure 30° with horizontal, then the height of the wire is

(1) 23 m (2) 18 m (3) 28 m (4) 25.5 m

30. The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45° and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is equal to

(1) 12.12 m (2) 14.14 m (3) 16.16 m (4) 18.18 m

7. MENSURATION

1. The curved surface area of a right circular cone of height 15cm and base diameter 16cm is

   (1) $60\pi\,\text{cm}^2$ (2) $68\pi\,\text{cm}^2$ (3) $120\pi\,\text{cm}^2$ (4) $136\pi\,\text{cm}^2$

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is

   (1) $4\pi r^2$ sq. units (2) $6\pi r^2$ sq. units (3) $3\pi r^2$ sq. units (4) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13cm will be

   (1) 12 cm (2) 10 cm (3) 13cm (4) 5cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

   (1) $1:2$ (2) $1:4$ (3) $1:6$ (4) $1:8$

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

   (1) $\frac{9\pi h^2}{8}$ sq.units (2) $24\pi h^2$ sq.units

   (3) $\frac{8\pi h^2}{9}$ sq.units (4) $\frac{56\pi h^2}{9}$ sq.units

6. In a hollow cylinder, the sum of the external and internal radii is 14cm and the width is 4cm. If its height is 20cm, then the volume of the material in it is

   (1) $5600\pi\text{ cm}^3$ (2) $1120\pi\,\text{cm}^3$

   (3) $56\pi\text{ cm}^3$ (4) $3600\pi\text{ cm}^3$

7. If the radius of the base of a cone is tripled and the height is doubled, then the volume is

   (1) made 6 times (2) made 18 times (3) made 12 times (4) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.

   (1) π (2) 4π (3) 3π (4) 2π

9. A solid sphere of radius $x\,\text{cm}$ is melted and cast into a shape of a solid cone of same radius. The height of the cone is

   (1) $3x$ cm (2) x cm (3) $4x$ cm (4) $2x$ cm

10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm, then the volume of the frustum is

    (1) $3328\pi\text{ cm}^3$ (2) $3228\pi\text{ cm}^3$

    (3) $3240\pi\text{ cm}^3$ (4) $3340\pi\text{ cm}^3$

11. A shuttle cock used for playing badminton has the shape of the combination of

    (1) a cylinder and a sphere (2) a hemisphere and a cone

    (3) a sphere and a cone (4) frustum of a cone and a hemisphere

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

(1) $2:1$ (2) $1:2$ (3) $4:1$ (4) $1:4$

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1cm and height 5cm is

(1) $\frac{4}{3}\pi$ (2) $\frac{10}{3}\pi$ (3) 5π (4) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1:2$, then $r_2 : r_1$ is

(1) $1:3$ (2) $1:2$ (3) $2:1$ (4) $3:1$

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is

(1) $1:2:3$ (2) $2:1:3$ (3) $1:3:2$ (4) $3:1:2$

16. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(1) $60\pi\,\text{cm}^2$ (2) $66\pi\,\text{cm}^2$ (3) $120\pi\,\text{cm}^2$ (4) $136\pi\,\text{cm}^2$

17. If S_1 denotes the total surface area of a sphere of radius r and S_2 denotes the total surface area of a cylinder of base radius r and height $2r$, then

(1) $S_1 = S_2$ (2) $S_1 > S_2$ (3) $S_1 < S_2$ (4) $S_1 = 2S_2$

18. The ratio of the volumes of two spheres is 8:27. If r and R are the radii of spheres respectively, then $(R-r):r$ is

(1) $1:2$ (2) $1:3$ (3) $2:3$ (4) $4:9$

19. The radius of a wire is decreased to one-third of the original. If volume remains the same, then the length will be increased _____ of the original.

(1) 3 times (2) 6 times (3) 9 times (4) 27 times

20. The height of a cone is $60\ cm.$ A small cone is cut off at the top by a plane parallel to the base and its volume is $\left(\frac{1}{64}\right)^{th}$ the volume the original cone. The height of the smaller cone is

(1) 45cm (2) 30cm (3) 15cm (4) 20cm

21. A solid frustum is of height 8 cm. If the radii of its lower and upper ends are 3 cm and 9 cm respectively, then its slant height is

(1) 15cm (2) 30cm (3) 10cm (4) 20cm

22. A solid is hemispherical at the bottom and conical above. If the curved surface areas of the two parts are equal, then the ratio of its radius and the height of its conical part is

(1) $1:3$ (2) $1:\sqrt{3}$ (3) $1:1$ (4) $\sqrt{3}:1$

23. The material of a cone is converted into the shape of a cylinder of equal radius. If the height of the cylinder is 5 cm, then height of the cone is

(1) 10cm (2) 15cm (3) 18cm (4) 24cm

24. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3.. The ratio of diameter to its height is

(1) $3:7$ (2) $7:3$ (3) $6:7$ (4) $7:6$

25. When Karuna divided surface area of a sphere by the sphere's volume, he got the answer as $\frac{1}{3}$.

What is the radius of the sphere?

(1) 24cm (2) 9cm (3) 54cm (4) 4.5cm

26. A spherical steel ball is melted to make 8 new identical balls. Then the radius each new ball is how much times the radius of the original ball?

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{8}$

27. A semicircular thin sheet of a metal of diameter 28cm is bent and an open conical cup is made. What is the capacity of the cup?

(1) $\left(\frac{100}{3}\right)\sqrt{3}\text{ cm}^3$ (2) $300\sqrt{3}\text{ cm}^3$

(3) $\left(\frac{700}{3}\right)\sqrt{3}\text{ cm}^2$ (4) $\left(\frac{1078}{3}\right)\sqrt{3}\text{ cm}^3$

28. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of wood wasted is

(1) 45% (2) 56% (3) 67% (4) 75%

29. A cylinder having radius 1m and height 5m is completely filled with milk. In how many conical flasks can this milk be filled if the flask radius and height is 50cm each?

(1) 50 (2) 500 (3) 120 (4) 160

30. A floating boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by $1cm$ when a man gets into it. The mass of the man is (density of water is 1000 kg/m^3)

(1) 50kg (2) 60 kg (3) 70 kg (4) 80 kg

8. STATISTICS AND PROBABILITY

1. Which of the following is not a measures of dispersion?

   (1) Range (2) Standard deviation (3) Arithmetic mean (4) Variance

2. The range of the data 8, 8, 8, 8, 8. . . 8 is

   (1) 0 (2) 1 (3) 8 (4) 3 3

3. The sum of all deviations of the data from its mean is

   (1) Always positive (2) always negative (3) zero (4) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

   (1) 40000 (2) 160900 (3) 160000 (4) 30000

5. Variance of first 20 natural numbers is

   (1) 32.25 (2) 44.25 (3) 33.25 (4) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5, then the new variance is

   (1) 3 (2) 15 (3) 5 (4) 225

7. If the standard deviation of x, y, z is p, then the standard deviation of $3x+5, 3y+5, 3z+5$ is

   (1) $3p+5$ (2) $3p$ (3) $p+5$ (4) $9p+15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is

   (1) 3.5 (2) 3 (3) 4.5 (4) 2.5

9. Which of the following is incorrect?

   (1) $P(A) > 1$ (2) $0 \le P(A) \le 1$ (3) $p(\phi) = 0$ (4) $P(A) + P(\overline{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

    (1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$ (3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

    (1) $\frac{3}{10}$ (2) $\frac{7}{10}$ (3) $\frac{3}{9}$ (4) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is

    (1) 2 (2) 1 (3) 3 (4) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

    (1) 5 (2) 10 (3) 15 (4) 20

14. If a letter is chosen at random from the English alphabets $\{a,b,......z\}$, then the probability that the letter chosen precedes x

(1) $\frac{12}{13}$ (2) $\frac{1}{13}$ (3) $\frac{23}{26}$ (4) $\frac{3}{26}$

15. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?

(1) $\frac{1}{5}$ (2) $\frac{3}{10}$ (3) $\frac{2}{3}$ (4) $\frac{4}{5}$

16. The range of first 10 prime numbers is

(1) 9 (2) 20 (3) 27 (4) 5

17. If the smallest value and co-efficient of range of a data are 25 and 0.5 respectively. Then the largest value is

(1) 25 (2) 75 (3) 100 (4) 12.5

18. If the observations $1, 2, 3, \ldots 50$ have the variance V_1 and the observations $51, 52, 53, \ldots 100$ have the variance V_2 then $\frac{V_1}{V_2}$ is

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) 0

19. If the standard deviation of a variable x is 4 and if $y = \frac{3x+5}{4}$, then the standard deviation of y is

(1) 4 (2) 3.5 (3) 3 (4) 2.5

20. If the data is multiplied by 4, then the corresponding variance is get multiplied by

(1) 4 (2) 16 (3) 2 (4) None

21. If the co-efficient of variation and standard deviation of a data are 35% and 7.7 respectively then the mean is

(1) 20 (2) 30 (3) 25 (4) 22

22. The batsman A is more consistent than batsman B if

(1) $\text{C.V of A} > \text{C.V of B}$ (2) $\text{C.V of A} < \text{C.V of B}$

(3) $\text{C.V of A} = \text{C.V of B}$ (4) $\text{C.V of A} \geq \text{C.V of B}$

23. If an event occurs surely, then its probability is

(1) 1 (2) 0 (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

24. A letter is selected at random from the word 'PROBABILITY'. The probability that it is not a vowel is

(1) $\frac{4}{11}$ (2) $\frac{7}{11}$ (3) $\frac{3}{11}$ (4) $\frac{6}{11}$

25. In a competition containing two events A and B, the probability of winning the events A and B are $\frac{1}{3}$ and $\frac{1}{4}$ respectively and the probability of winning both the events is $\frac{1}{12}$. The probability of winning only one event is

(1) $\frac{1}{12}$ (2) $\frac{5}{12}$ (3) $\frac{1}{12}$ (4) $\frac{7}{12}$

26. A number x is chosen at random from $-4, -3, -2, -1, 0, 1, 2, 3, 4$. The probability that $|x| \leq 3$ is

(1) $\frac{3}{9}$ (2) $\frac{4}{9}$ (3) $\frac{2}{9}$ (4) $\frac{7}{9}$

27. If the probability of non-happening of an event is q, then the probability of happening of the event is

(1) $1-q$ (2) q (3) $\frac{q}{2}$ (4) $2q$

28. In one thousand lottery tickets, there are 50 prizes to be given. The probability of Mani winning a prize who bought one ticket is

(1) $\frac{1}{50}$ (2) $\frac{1}{100}$ (3) $\frac{1}{1000}$ (4) $\frac{1}{20}$

29. When three coins are tossed, the probability of getting the same face on all the three coin is

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$ (3) $\frac{3}{8}$ (4) $\frac{1}{3}$

30. A box contains some milk chocolates and some coco chocolates and there are 60 chocolates in the box. If the probability of taking a milk chocolate is $\frac{2}{3}$ then the number of coco chocolates is

(1) 40 (2) 50 (3) 20 (4) 30

ONE MARK ANSWERS

1. RELATIONS AND FUNCTIONS

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(3)**	**(3)**	**(1)**	**(2)**	**(3)**	**(4)**	**(3)**	**(1)**	**(3)**	**(3)**	**(1)**	**(4)**	**(3)**	**(2)**	**(4)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(1)**	**(1)**	**(3)**	**(4)**	**(2)**	**(2)**	**(1)**	**(3)**	**(2)**	**(4)**	**(4)**	**(1)**	**(2)**	**(2)**	**(2)**

2. NUMBERS AND SEQUENCES

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Answer	**(3)**	**(1)**	**(2)**	**(3)**	**(4)**	**(1)**	**(4)**	**(3)**	**(1)**	**(3)**	**(3)**	**(4)**	**(2)**	**(2)**	**(3)**	**(2)**	**(1)**
Q.No.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	--
Answer	**(1)**	**(2)**	**(2)**	**(1)**	**(4)**	**(3)**	**(2)**	**(3)**	**(1)**	**(1)**	**(3)**	**(3)**	**(3)**	**(2)**	**(1)**	**(3)**	

3. ALGEBRA

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(4)**	**(4)**	**(2)**	**(1)**	**(2)**	**(3)**	**(4)**	**(2)**	**(3)**	**(3)**	**(2)**	**(1)**	**(2)**	**(4)**	**(2)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(2)**	**(4)**	**(2)**	**(3)**	**(1)**	**(2)**	**(3)**	**(2)**	**(2)**	**(3)**	**(2)**	**(2)**	**(1)**	**(2)**	**(3)**
Q.No.	31	32	33	34	35	36	37	38	39	40	--	--	--	--	--
Answer	**(1)**	**(1)**	**(3)**	**(2)**	**(2)**	**(4)**	**(1)**	**(1)**	**(1)**	**(3)**	--	--	--	--	--

4. GEOMETRY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(3)**	**(2)**	**(4)**	**(1)**	**(4)**	**(1)**	**(2)**	**(3)**	**(1)**	**(4)**	**(2)**	**(2)**	**(2)**	**(4)**	**(1)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(2)**	**(3)**	**(3)**	**(3)**	**(4)**	**(1)**	**(2)**	**(3)**	**(2)**	**(2)**	**(1)**	**(3)**	**(3)**	**(2)**	**(1)**

5. CO-ORDINATE GEOMETRY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(2)**	**(1)**	**(2)**	**(3)**	**(3)**	**(4)**	**(2)**	**(2)**	**(1)**	**(3)**	**(3)**	**(1)**	**(2)**	**(1)**	**(2)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(3)**	**(3)**	**(1)**	**(4)**	**(1)**	**(3)**	**(1)**	**(2)**	**(1)**	**(2)**	**(3)**	**(2)**	**(4)**	**(3)**	**(3)**

6. TRIGONOMETRY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(2)**	**(4)**	**(2)**	**(1)**	**(2)**	**(2)**	**(1)**	**(3)**	**(2)**	**(4)**	**(2)**	**(2)**	**(4)**	**(2)**	**(1)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(3)**	**(4)**	**(2)**	**(1)**	**(2)**	**(2)**	**(3)**	**(2)**	**(3)**	**(4)**	**(2)**	**(1)**	**(3)**	**(1)**	**(2)**

7. MENSURATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(4)**	**(1)**	**(1)**	**(2)**	**(3)**	**(2)**	**(2)**	**(3)**	**(3)**	**(1)**	**(4)**	**(1)**	**(1)**	**(2)**	**(4)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(4)**	**(3)**	**(1)**	**(3)**	**(3)**	**(3)**	**(2)**	**(2)**	**(2)**	**(2)**	**(3)**	**(4)**	**(4)**	**(3)**	**(2)**

8. STATISTICS AND PROBABILITY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**(3)**	**(1)**	**(3)**	**(2)**	**(3)**	**(4)**	**(2)**	**(1)**	**(1)**	**(2)**	**(2)**	**(2)**	**(3)**	**(3)**	**(4)**
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**(3)**	**(2)**	**(1)**	**(3)**	**(2)**	**(4)**	**(2)**	**(1)**	**(2)**	**(2)**	**(4)**	**(1)**	**(1)**	**(2)**	**(3)**